Volume II. *September–October, 1894.* *Number 2.*

THE

PHYSICAL REVIEW.

ON THE SOLUTION-TENSION OF METALS.

By Harry C. Jones.

FROM van't Hoff's theory of osmotic pressure, the molecules of dissolved substances exert a certain pressure in the solvent in which they are dissolved. Nernst[1] has pointed out that in order that a substance may dissolve in a solvent, it must have a certain "expansive force" to drive the molecules into the space where they will exert an osmotic pressure.

He has extended the idea of "solution-tension" also to the pure metals. When a metal is brought in contact with a solution of its salt, a very small part of the metal is conceived to be capable of taking plus electricity from the remainder of the metal and of passing into solution as ions, when the osmotic pressure of the metallic ions of the salt solution is not already greater than the solution-tension of the metal. To consider more closely concrete cases in the light of the suggestion of Nernst, let us consider a metal immersed in a solution of one of its salts. Let the solution-tension of the metal be T and the osmotic pressure of the metallic ions of the dissolved salt p. If the solution-tension be greater than the osmotic pressure, $T > p$, then a small part of the metal will pass into solution as ions. These ions will carry off a positive charge of electricity from the metal into the solution. The metal will then contain a corresponding negative charge, and the solution will

[1] Nernst, Zeits. Phys. Chem., 4, 129.

be positive with respect to the metal. The negative electricity on the surface of the metal will attract electrostatically the positive ions in the solution, and on the surface of contact between the metal and solution there will be formed a double electrical layer. The existence of such a double layer has been shown to be probable by Helmholtz,[1] who thinks that such must always exist on the bounding surface of two conductors at different potentials.

Under these conditions, where the metal is negative and the solution positive, the action of the double layer would be to drive the metallic ions out of solution and on to the metal, where they would lose their charge and pass over into the metallic condition. The attraction between this double layer, which tends to drive metallic ions out of solution, works against the solution-tension of the metal. Equilibrium will be established when these two contrary forces are equal, when in any given time as many metallic ions will be driven into the solution due to the solution-tension of the metal as will separate from the solution due to the electrostatic attraction of the double electrical layer. The difference in the potential of this double layer depends in part on the nature of the metal, but also in part on the concentration of the metallic ions in the solution, and therefore upon the osmotic pressure of the metallic ions in the solution. The action of the above two forces gives rise to an electromotive force between the metal and the solution in which it is immersed. Since $T > p$, the solution will be positive with respect to the metal, and the electromotive force will exist in the sense of producing a current from the metal to the liquid.

If $T < p$, the reverse will take place. Metallic ions will separate from the solution as metal on the immersed metal, which will become positive with respect to the solution. The positive charge on the metal will attract electrostatically the negative ions in the liquid, and an electrical double layer will be formed here, but in the reverse sense to that discussed above. Metallic ions will separate from the solution until the electrostatic repulsion of such ions by the positive metal is equal to the osmotic pressure tending to force them out of solution. The result of the action of these

[1] Helmholtz, Wied. Ann., 7, 337.

forces is to produce an electromotive force between the metal and solution. But since $T < p$, the metal is positive with respect to the solution, and the electromotive force will exist in the sense of generating a current from the solution to the metal, when the necessary connections are established.

This explanation of the source of the electromotive force, which exists between a metal and solutions of its salts, based on the relation between the osmotic pressure of such solutions, and the solution-tension of the metal, has been furnished by Nernst,[1] who has developed it essentially as given above. He has also called attention to the fact that the amount of metal which would pass into solution as ions under the condition $T > p$ is in general extremely small because of the great electrostatic capacity of the ions.

Ostwald[2] has developed a formula for calculating the solution-tension of metals, knowing the difference in potential between the metal and the solution of its salt in which the metal is immersed, and the osmotic pressure of the salt solution, thus:

Let a metal be immersed in a solution of one of its salts. By passing $N_e E_0$ (N_e = valence of the kation, $E_0 = 96540$ Coul.) of electricity through the metal, a gram-atomic weight of the metal is dissolved. Osmotic energy is transformed into electrical. Let P be the solution-tension of the metal, and p the osmotic pressure of the metallic ions in the solution. The isothermal change of the volume energy consists of two parts, the values of which are Pv and $\int_P^p v dp$. In terms of the gas law,

$$Pv = RT, \text{ and } \int_P^p v dp = RT ln \frac{p}{P} = -RT ln \frac{P}{p}.$$

The two together,

$$RT\left(1 - ln \frac{P}{p}\right).$$

But this same equation which applies to gases also holds for solutions where osmotic pressure is substituted for gas pressure.

[1] Nernst, Zeits. Phys. Chem., 4, 150. [2] Ostwald, Lehrb., Bd. II., 851.

But since the volume energy has decreased, the value is to be taken as negative, and the alteration of the volume energy then is

$$RT\left(ln\frac{P}{p}-1\right).$$

The electrical work is $N_eE_0\pi$, which is just equal to the change in the volume energy. π = the difference in potential between the metal and the solution. Equating these two equal expressions, we have

$$\pi N_eE_0=RT\left(ln\frac{P}{p}-1\right),$$

from which

$$\pi=\frac{RT}{N_eE_0}\left(ln\frac{P}{p}-1\right).$$

The electrolytic solution-tension seemed from what was hitherto known to be a constant for each metal, increasing in general with increase in temperature [1] as the vapor-tension and solubility increase with the temperature.

Ostwald [2] has calculated the solution-tension of a number of the metals from the formula which he developed. In these calculations the potential differences between metals and solutions of their salts as measured by Wright and Thompson,[3] and by F. Braun,[4] were employed. The difference in potential between a metal and a solution of one of its salts depends upon the solution-tension of the metal on the one hand, and the osmotic pressure of the metallic ions in the solution acting against the solution-tension on the other.

In the formula $\pi=\frac{RT}{N_eE_0}\left(n\frac{P}{p}-1\right)$, in addition to the constant there are three values involved, the difference in potential between the metal and the solution of its salt in which it is immersed $=\pi$, the osmotic pressure of the solution $=p$, and the solution-tension of the metal $=P$. But the difference in potential between the metal and a definite concentration of its salt solution must be

[1] Nernst, Zeits. Phys. Chem., 4, 172. [2] Ostwald, Lehrb., Bd. II., 946.
[3] Wright and Thompson, Phil. Mag. [5], 19, 209.
[4] Braun, Wied. Ann., 16, 561, and 17, 593.

determined, and also the osmotic pressure of a solution of the same concentration, in order that the solution-tension may be calculated. Normal solutions containing a gram-atomic weight of the metal in a liter were employed. Such solutions of normally dissociated salts are about 80 per cent dissociated, and have an osmotic pressure of about 22 atmospheres. The solution-tension of the following metals was calculated from the formula given above. If in this we substitute the value of $\frac{RT}{E_0}=0.058$, passing from the natural to the Briggean logarithms, we obtain

$$\log P=\frac{\pi N_e}{0.058}+\log p+0.434.$$

The solution tension, in atmospheres, as calculated by Ostwald from this formula, using the measurements of potential differences between metals and solutions of their salts made by Wright and Thompson, and Braun, are given below: —

Magnesium 10^{44}	Cadmium 10^{7}	Copper 10^{-19}
Zinc 10^{18}	Iron 10^{3}	Mercury 10^{-15}
Aluminium 10^{13}	Lead 10^{-2}	Silver 10^{-15}

From the above figures, the metals show striking differences as to their solution-tension in water solutions of their salts. The calculated values are in general either very great or very small. Of the above metals, iron and lead have intermediate values. These values can be taken as expressing the approximate solution-tensions of the above metals.

The main point in mind when the work described in this paper was undertaken, was the determination of the electromotive force between solutions of salts in water and solutions of the same salts and of the same concentrations in other solvents, using the same metal as the kation of the salt as electrodes. Knowing the dissociation of a salt in water, it seemed probable that the dissociation in other solvents could be calculated from the electromotive force. But this calculation would be based on the assumption that the solution-tension of a metal is a constant and the same constant for the different solvents in which its salts were dissolved.

But little is known at present of the dissociation of salts in solvents other than water, and the above method based on the measurements of electromotive force seemed to promise much in this direction. The dissociation as calculated from electromotive force could then be compared with that calculated from conductivity and "migration velocities" of the ions in the different solvents, when such determinations shall have been made. Indeed, some work on the conductivity of alcoholic solutions of salts has already been carried out.[1]

Silver nitrate solutions alone have been carefully studied thus far in my work. A tenth-normal solution of this salt in water and a tenth-normal solution in ethyl alcohol were prepared. These were placed respectively in the glass vessels, *A* and *B*, Fig. 1. Both of these vessels were connected with the middle vessel, *C*, which was filled with the water solution of silver nitrate of the same strength as that in vessel *A*, by means of the side tubes, as shown in Fig. 1. The ends, *HH*, of these side tubes were closed loosely with a roll of filter paper, to diminish the diffusion from one vessel to the other, and the consequent mixing of the solutions. The liquid in the three vessels was then continuously connected, and at the same time could not pass rapidly from one to the other. Into the solutions in *A* and *B*, were dipped the silver electrodes *EE*. The silver wires, *FF*, were attached to the silver electrodes and surrounded by the small glass tubes *gg*, where they passed through the corks *DD*. The upper and lower ends of each glass tube were fused to the wire by means of fusing glass. The use of the short glass tube was to make closer contact in passing through the cork than would have been possible with the metallic wire alone.

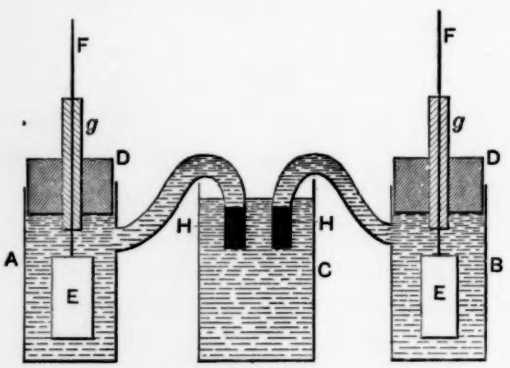

Fig. 1.

[1] Völlmer, Dissertation, Halle.

The silver electrodes were cut from thin sheet silver and carefully cleaned. They were then covered with a layer of silver deposited electrolytically from a solution of potassium-silver cyanide. In order to secure a uniform deposit, the solution of the double cyanide must be very dilute, and the current not stronger than that of one Leclanché element. About a half-dozen such electrodes were plated together, all in connection with the same binding-screw. But even when thus plated together, the electrodes were not always exactly constant, the one against the other. They however became constant to within a few ten-thousandths of a volt if allowed to stand in combination with each other for some time in a dilute solution of silver nitrate in water. Such a solution of silver nitrate contains silver ions, and any small differences in potential between the electrodes immersed in this solution would gradually disappear.

At first measurements of electromotive force between one-tenth normal solutions in alcohol and in water were made, and then more dilute solutions were investigated. The same concentration of solution in both solvents was always employed. The following solvents were used for silver nitrate; water, ethyl alcohol, methyl alcohol, and acetone. The first series of measurements was made between water solutions and ethyl alcohol solutions. All measurements of potential differences were made by means of the most sensitive form of the Lippmann electrometer,[1] consisting of the upright capillary tube of very small diameter. Readings to 0.0002 volt could be made with a fair degree of accuracy. All measurements were referred to a Clark element which had been compared with a standard Clark. The usual volt element was prepared. This was connected against a Leclanché element through a convenient form of resistance box, and the whole with the electrometer. The point of equilibrium was then determined on the box. Then the Clark minus the volt, *i.e.* the difference, was connected against the Leclanché, and the equilibrium determined on the box. This gave data for calculating the real value of the supposed volt, knowing the temperature, and therefore the electromotive force of the Clark. Two determinations of the value of the supposed volt

[1] Ostwald, Lehrb., Bd. II., 924.

made some weeks apart showed it to be respectively 0.9955 and 0.9946 volt. Knowing the value of the Clark, and also of the supposed volt, their difference is known. In order that the very sensitive electrometer might be used, the Leclanché was closed through about 2000 ohms resistance, and then the difference between the Clark and volt was brought to equilibrium on the box against the Leclanché with this extra resistance. Electromotive forces were then measured on the box against the Leclanché with the above resistance. The Leclanché was never closed except during the short intervals in which measurements were actually being made, and was compared from day to day against the difference between the Clark and volt to detect any changes. The Leclanché was found to become weaker, but changed very slowly.

The following table contains the measurements of electromotive force between water solutions and ethyl alcohol solutions. The first column contains the concentrations of the water solutions in terms of normal solutions; the second, those of the alcohol which are the same; the third, the first series of measurements of electromotive force; the fourth, the second series of such measurements. The second series of measurements was made with newly prepared solutions.

TABLE I.

Concentrations in water solutions. Normal.	Concentrations in ethyl alcohol solutions. Normal.	Electromotive force. 1st series.	Electromotive force. 2d series.
0.1	0.1	0.0956	0.0951
0.05	0.05	0.1055	0.1045
0.01	0.01	0.1185	0.1199
0.005	0.005	0.1274	0.1284
0.001	0.001	0.1309	0.1300

The *alcoholic solutions* were always *positive* against the water solutions, and to the extent given by the above results. The difference increases with the dilution, as seen from the results. Since this work was begun, a publication has appeared from Mr. Adolfo Campetti (Dissertation, Turin), describing measurements

of the electromotive force between alcohol and aqueous solutions of ammonium, lithium, calcium, copper, zinc, and cadmium chlorides, and zinc and cadmium iodides. He also found that the alcoholic solutions were positive against the water solutions, but does not seem to have seen any special significance in his results as far as the determination of the solution-tension of metals is concerned.

In order that the alcoholic solutions should be positive against the water solutions, silver ions must separate from the alcoholic solutions as metal on the electrode. Whether this takes place depends, as we have seen, on the relation between the osmotic pressure of the metallic ions in the solution and the solution-tension of the metal. The water solutions are far more dissociated than the alcoholic and the osmotic pressure is therefore greater in the water solutions, yet ions separate from the alcoholic solution on the electrode when connected against the water solution of the same degree of concentration. This separation of ions takes place from the alcoholic solution which has *less* osmotic pressure; therefore, the force opposing this separation — the *solution-tension* — must be *less* in alcoholic solutions.

While but little is known in general of the dissociation of compounds in alcohol, yet a few substances have been studied by Völlmer[1] as to their conductivity, and among these is silver nitrate. Knowing the dissociation of silver nitrate in water and in alcoholic solutions of the same strength, it seems to be possible to calculate the relative value of the solution-tension of silver in water and in alcoholic solutions from the electromotive force existing between the two.

Let π_1 be the electromotive force of the alcoholic solution in which the silver electrode is immersed; then,

$$\pi_1 = \frac{RT}{N_e E_0} \log \frac{p_1}{P_1},$$

where p_1 is the osmotic pressure of the alcoholic solution, and P_1 the solution tension of the silver in the alcoholic solu-

[1] Völlmer, Dissertation, Halle.

tion. Let π_2 be the electromotive force of the water solution; then

$$\pi_2 = \frac{RT}{NE_0} \log \frac{p_2}{P_2},$$

where p_2 is the osmotic pressure of the water solution of silver nitrate, and P_2 the solution-tension of the silver in the water solution. The electromotive force determined was the difference between π_1 and π_2.

$$\pi_1 - \pi_2 = \frac{RT}{N_e E_0} \log \frac{p_1}{P_1} - \frac{RT}{N_e E_0} \log \frac{p_2}{P_2};$$

but since $\frac{RT}{E_0} = 0.058$ volt, this becomes

$$\pi_1 - \pi_2 = \frac{0.058}{N_e} \log \frac{p_1}{P_1} - \frac{0.058}{N_e} \log \frac{p_2}{P_2}.$$

But N_e = the valence of the kation for silver = 1, we have then

$$\pi_1 - \pi_2 = 0.058 \left(\log \frac{p_1}{P_1} - \log \frac{p_2}{P_2} \right).$$

In the last formula all the values are known except P_1 and P_2, taking the dissociation of silver nitrate in alcohol as found by Völlmer.[1] By making $P_2 = 1$, the value of P_1 in terms of P_2 can be calculated

$$\log P_1 = -\left(\frac{\pi_1 - \pi_2}{0.058} \right) + \log \frac{p_1}{p_2}.$$

The conductivity of the following concentrations which lie within the range of my work has been determined, 0.025 and 0.0025 normal. The molecular conductivity as found by Völlmer in alcohol, for these concentrations, is 136.3 and 246.3 respectively. The greatest molecular lowerings found for silver nitrate in alcohol were for 0.00002445 normal, 350.0, and for 0.000007733 normal, 353.0, and these values can be regarded as very nearly the maximum for such solutions. The limit seems, however, not to have been quite reached, and the true limit found by Völlmer by extrapolation was 356. The limit found by Kohlrausch for water solutions of silver nitrate was $M\infty = 1080$. The value of $M\infty$ for this

[1] Völlmer, Dissertation, Halle.

and other salts in water, in comparison with the highest value found for alcohol, was about 3, with a mean value of 2.93. The value of $M\infty$ for water can be taken as representing complete dissociation; therefore the highest value found in alcohol would represent $\frac{1}{2.93}$ of complete dissociation if the "velocity of migration" of the ions was the same in alcohol as in water. Poisenilles has shown that the "internal friction" of alcohol is to water as 682 : 523. The above value $\frac{1}{2.93}$ must therefore be multiplied by $\frac{682}{523}$. $\frac{682}{2.93 \times 523} = \frac{1}{2.25}$ Therefore in the most dilute alcoholic solutions investigated by Völlmer only $\frac{1}{2.25}$ part of the molecules was dissociated. The true value of $M\infty$ for an alcoholic solution of silver nitrate corresponding to complete dissociation is therefore about $356 \times 2.25 = 800$.

The dissociation of silver nitrate in alcohol is then for the concentration 0.025, $\frac{136}{800} = 17$ per cent, and for the concentration 0.0025, $\frac{246}{800} = 31$ per cent.

The dissociation of silver nitrate in water for these concentrations as found by interpolation in the results of Kohlrausch is for the concentration 0.025, 90 per cent; and for the concentration 0.0025, 97 per cent. Inserting these values for the dissociation in the formula,

$$\log P_1 = -\left(\frac{\pi - \pi_2}{0.058}\right) + \log \frac{p_1}{p_2},$$

for p_1 the dissociation in the alcoholic solution, and for p_2 the dissociation in the water solution, we have all the data necessary for calculating the solution-tension P_1. The value of P_1 for the above two concentrations will be calculated, using for $(\pi_1 - \pi_2)$ the value found by interpolation in my measurements of the electromotive force between alcoholic and water solutions.

The value of $\pi_1 - \pi_2$ for the concentration 0.025 is 0.1135 v.
The value of $\pi_1 - \pi_2$ for the concentration 0.0025 is 0.1295 v.
The solution-tension P_1 calculated for the conc. 0.025 is 0.048.
The solution-tension P_1 calculated for the conc. 0.0025 is 0.053.

As has been seen, this calculation is based on the dissociation of alcoholic solutions as given by the work of Völlmer, following also his line of reasoning. If these dissociations for alcoholic solutions should be shown to contain any error, it will be perfectly simple to insert the true dissociation of such alcoholic solutions in the above formula, and recalculate the solution-tension of the metal in the solution.

The value of P_1 as calculated for the concentration 0.025 normal, agrees well with that calculated for one-tenth this concentration, using the dissociation values of Völlmer and Kohlrausch for the alcoholic and water solutions respectively, and my own measurements of the electromotive force of the solutions against each other. From these results the solution-tension of silver in alcoholic solutions of silver nitrate is only about one-twentieth of that in aqueous solutions of the same salt.

If we regard 356 as representing complete dissociation of silver nitrate in ethyl alcohol, then the calculated values of P_1 for the two concentrations are respectively 0.021 and 0.024, while $p_2 = 1$.

The increase in the electromotive force of the alcoholic solutions of silver nitrate against the water solutions, with increase in dilution, is due doubtless to the more rapid increase in the dissociation of the alcoholic solutions of the salt for the concentrations used.

It was desired to test the effect of changing the anion of the silver salt on the solution-tension of the metal in alcoholic solutions of the salt. To accomplish this, attempts were made to measure the electromotive force of alcoholic solutions of silver benzoate and of silver acetate against water solutions of the same concentrations, using silver electrodes as before. But on account of the slight solubility of these salts in ethyl alcohol, no satisfactory results could be obtained. The question, then, as to the effect of the anion of the silver salt on the solution-tension of the metal in alcoholic solutions of the salt remains at present entirely unsettled.

The electromotive force between 0.01 n. silver nitrate in water, and 0.01 n. silver nitrate in ethyl alcohol, is about 0.1190 volt. The electromotive force between two solutions of 0.01 n. silver nitrate in water or in alcohol would be of course zero. It is then

of interest to add varying amounts of the solution in one solvent to the solution in the other, and measure the potential difference against the solution in one or in the other solvent. Varying amounts of the water solution 0.01 n. were added to a constant amount of the 0.01 n. alcohol solution. Except for the slight change in volume, the mixtures remained then 0.01 n.; the difference in potential of these mixtures, against 0.01 n. solution of silver nitrate in water was then determined. The results are given in Table II. A = the amount of the alcoholic solu-

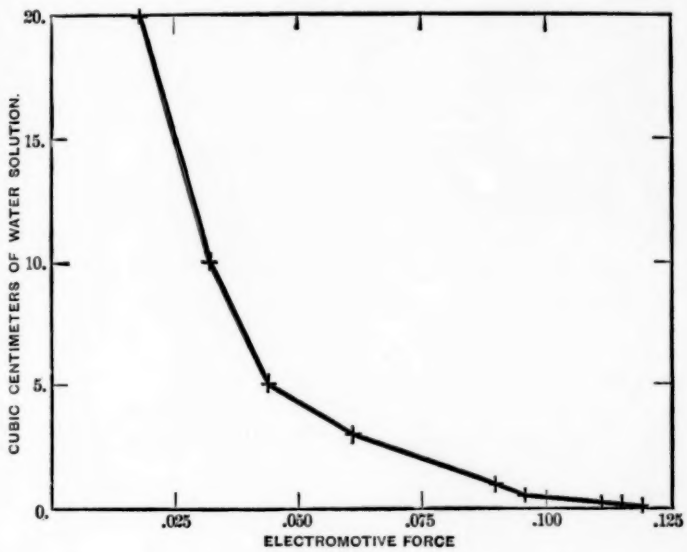

Fig. 2.

tion (10 c.c.), W = the different amounts of the water solution added to the 10 c.c. alcoholic solution. E = the electromotive force found.

These results are plotted as a curve, Fig. 2. The first measurement, where the largest amount of water was used, was omitted for convenience in selecting the scale for plotting the remaining results.

TABLE II.

A	*W*	*E*
10 c.c.	100 c.c.	0.0030
10 "	20 "	0.0179
10 "	10 "	0.0319
10 "	5 "	0.0438
10 "	3 "	0.0605
10 "	1 "	0.0901
10 "	0.5 "	0.0961
10 "	0.2 "	0.1110
10 "	0.1 "	0.1150
10 "	0.0 "	0.1192

Solutions in Methyl Alcohol.

The methyl alcohol used, like the ethyl alcohol, was purified and freed from water, and the boiling-point determined. The electromotive force of solutions of silver nitrate in methyl alcohol against solutions of the same concentration in water, was determined in all respects as with ethyl alcohol. Below are the results tabulated as those with ethyl alcohol.

TABLE III.

Concentrations, normal in water solutions.	Concentrations, normal in methyl alcohol solutions.	Electromotive force. 1st series.	Electromotive force. 2d series.
0.1	0.1	0.0906 volt	0.0892 volt
0.05	0.05	0.1000	—
0.01	0.01	0.1131	0.1117
0.005	0.005	0.1180	—
0.001	0.001	0.1230	0.1219

The methyl alcohol solutions were always positive against the water solutions of the same concentration and to the amounts given in the above table. The methyl alcohol solutions are, however, somewhat less positive against the water solutions than the ethyl alcohol solutions, as seen by comparing the electromotive force values in the above table with those in Table I. This is what

might be expected from the composition of the two alcohols with respect to water.

When it was found that the ethyl alcohol solutions gave greater potential differences against water solutions than the corresponding methyl alcohol solutions, it was supposed that the ethyl alcohol solutions would be positive against methyl alcohol solutions of the same concentration. The reverse was found to be true. For the concentrations used, the methyl alcoholic silver nitrate was always positive against the ethyl, and to the amounts given below.

TABLE IV.

Concentrations, normal in ethyl alcohol solutions.	Concentrations, normal in methyl alcohol solutions.	Electromotive force. Methyl alcohol positive.
0.1	0.1	0.0058 volt
0.05	0.05	0.0093
0.01	0.01	0.0098
0.005	0.005	0.0058
0.001	0.001	0.0097

This seems to indicate an electromotive force at the contact between the water and alcohol solutions, and that this differs in amount for the contact between water and ethyl alcohol, and water and methyl alcohol solutions. It appears to be possible to calculate the electromotive force on the contact between any two different concentrations of the same electrolyte in the same solvent, knowing the velocity of migration of the anion and kation in that solvent, and the dissociation of the two solutions. But here we have to do with two solvents and the calculation of the electromotive force at the contact is probably at best only roughly approximate. Ostwald[1] has deduced a formula for calculating the electromotive force on the contact of two different concentrations of the same electrolyte, from the work of Nernst,[2] thus: π = the electromotive force to be calculated, p_1 = the osmotic pressure of both ions in the more concentrated solution, p_2 = the osmotic pressure in the more dilute, u = the velocity of migration of the

[1] Ostwald, Lehrb., Bd. II., 843.

[2] Nernst, Zeits. Phys. Chem., 4, 129.

kation, v = that of the anion. By passing E_0 electricity through the contact of the two solutions, $\frac{u}{u+v}$ gram-equivalent of the kation will migrate in the positive sense, and $\frac{v}{u+v}$ of the anion in the negative. This amount of the kation $\frac{u}{u+v}$ passes from where the pressure is p_1 to where the pressure is p_2, and the corresponding osmotic energy is $\frac{u}{u+v} RTln\frac{p_1}{p_2}$. The anion has meanwhile passed from a lower pressure into a higher, supposing the stream to pass from the more concentrated to the more dilute solution and the energy spent is $\frac{v}{u+v} RTln\frac{p_1}{p_2}$. The gain in energy is therefore the difference between the above values = $\frac{u-v}{u+v} RTln\frac{p_1}{p_2}$. Placing the electrical energy πE_0 equal to the osmotic, we have

$$\pi = \frac{u-v}{u+v}\frac{RT}{E_0} ln\frac{p_1}{p_2},$$

$$= \frac{u-v}{u+v} 0.0002\ T \log\frac{p_1}{p_2}.$$

If the velocity of migration of the ions were the same in alcohol as in water solutions, the electromotive force π on the contact for 0.025 normal silver nitrate would be apparently about 0.002 volt. Another term, u_1v_1, for the velocity of migration of the kation and anion respectively, in the alcoholic solutions can be inserted in the above formula, when these shall have been more accurately determined. The value found for π would then probably be too small to account for the difference in potential between ethyl and methyl alcohol solutions of silver nitrate when measured against water solutions, and when measured directly against each other.

From the results, the solution-tension of silver in methyl alcohol solutions of silver nitrate, as in ethyl alcohol solutions, is less than in water solutions. Since but little is known of the dissociation of salts in methyl alcohol, the value of the solution-tension of metallic silver in methyl alcohol cannot at present be calculated.

Solutions of Silver Nitrate in Acetone.

Silver nitrate was dissolved in acetone which had been purified and whose boiling-point had been determined. A standard solution was prepared as with methyl and ethyl alcohol solutions, and from this the dilute solutions were made. A few measurements were made of acetone solutions against water solutions of the same concentrations.

TABLE V.

Concentrations, normal in water solutions.	Concentrations, normal in acetone solutions.	Electromotive force. 1st series.	Electromotive force. 2d series.
0.01	0.01	0.1521 volt	0.1512 volt
0.005	0.005	0.1692	——
0.001	0.001	0.1814	0.1829

The acetone solutions were always positive. Since the dissociation of silver nitrate in acetone is not known, it is not possible at present to calculate the solution-tension of silver in acetone solutions of silver nitrate. But the line of reasoning which was employed in the case of ethyl alcohol solutions would lead to the conclusion that the solution-tension is also *less* in acetone than in water solutions.

The electromotive force here, as with methyl and ethyl alcohol, increases with increase in the dilution, but somewhat faster with acetone than with the alcohols. This probably means that the dissociation of the salt in acetone increases slightly faster with respect to the water solutions for these concentrations than the dissociation of the salt in the alcohols.

I also tried to make a series of measurements of the electromotive force of alcoholic solutions of zinc chloride against water solutions. Mr. Campetti[1] described such measurements. I used different forms of zinc electrodes, — amalgamated zinc, polished zinc, and electrodes covered with zinc deposited electrolytically. The results which were obtained for one-tenth normal solutions in

[1] Campetti, Dissertation, Torino.

the two solvents, water and alcohol, varied from 0.15 to 0.25 volt. But the *alcoholic solutions* were *always positive* against the *water*. If the electrodes were constant when introduced respectively into the water and alcoholic solutions, after standing a short time in the solutions they were no longer constant against each other. A white precipitate was frequently observed in small amounts in the alcoholic solution after standing for some time in contact with the zinc electrode, indicating some action between the solution and the electrode. The electromotive force frequently changed very considerably on allowing the two solutions to stand for some time in contact with the electrodes. I hope to continue the work with other metals.

The point of chief interest brought out in connection with this investigation is that the *solution-tension of silver is not a constant for all solvents of its salts*, but depends upon the nature of the solvent in which its salts are dissolved. The same fact will probably be found true for other metals.

In conclusion I wish to express my sincere thanks to Professor Ostwald for valuable suggestions during the progress of this work, which was carried out in his laboratory.

LEIPZIG, PHYS-CHEMICAL LABORATORY,
March, 1894.

ON ELECTRIC STRENGTH.

By G. W. Pierce.

IN a paper by Dr. Macfarlane and the writer, published in the *Physical Review*, November–December, 1893, were given the results of a series of experiments on the electric strength of paraffined paper, beeswaxed paper, and kerosene oil. The apparatus consisted simply in two discharging tables connected in parallel with the poles of a Toepler-Holtz machine. Between a pair of brass disks about four inches in diameter, on one of the tables, was placed the solid or liquid to be tested. The system was gradually charged, and the pair of similar disks on the other table were slowly separated, causing the air gap between them to grow larger and larger until the spark ceased passing through the air, and broke down the insulator tested.

The air thickness was obtained by the reading on a micrometer-millimeter scale carried by one of the tables. The thickness of the dielectric near the hole made by the discharge was measured by a spherometer.

In the experiments described below, as in the earlier experiments, I have compared the different dielectrics with air, and have then attempted to reduce them to absolute measure by taking from the papers of Dr. Macfarlane,[1] Mr. Steinmetz,[2] and M. Baille[3] the difference of potential corresponding to the equivalent air values obtained.

Table I. gives the results obtained at different temperatures for melted paraffin, of which the melting-point is 52° C. In most of

[1] Trans. Royal Soc. Edinb., Vol. 28, p. 633, and Vol. 29, p. 561. Proc. R. S. E., Vol. 10, p. 555; Phil. Mag., December, 1880; Trans. Amer. Inst. El. Eng., March, 1893.

[2] Trans. Amer. Inst. El. Eng., February, 1893.

[3] Thomson's Recent Researches in Electricity and Magnetism, p. 70.

the tests it appears that raising the temperature strengthens the dielectric. I was unable to determine this point definitely.

Figure 1 represents the results graphically. Curve I. gives the relations between the thickness of the liquid and the corresponding thickness of air. Curve II. (using Baille's air equivalent), curve III. (using Macfarlane's air equivalent), and curve IV. (using Steinmetz's equivalent) give the difference of potential as a function of the length of the spark produced through the paraffin.

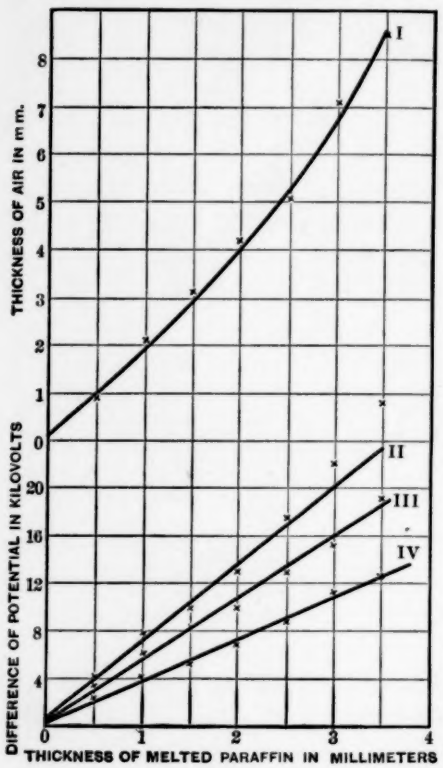

Fig. 1.

I also tested in the melted condition a paraffin of higher melting-point (76°–80° C.) and obtained the result that the electric strength is constant and somewhat greater than the 52° paraffin. Compare Table II. and Fig. 2 with Table I. and Fig. 1.

In order to test solid paraffin, I prepared a number of thin cakes by pouring the melted paraffin on the surface of hot mercury and allowing to cool, so that the cakes were of almost absolutely uniform thickness, and practically free from air bubbles. The paraffins used, with the exception of the 52° paraffin, were prepared by Drs. Bender and Hobein of Munich. In the case of the 52° paraffin (see Table I. and Fig. 3) the electrodes were the plates used in the experiments on paraffined and beeswaxed paper. I observed that with such a large surface of the dielectric exposed to the electrodes, the spark generally found a hole or defect to go through; and it is probable that the gradients of Table III. are somewhat

too small. To obviate this difficulty in the subsequent experiments, balls of 2.5 cm. diameter were used for the paraffin electrodes, the air electrodes being balls (Table IV.) or plates (Tables V. and VI.). In Table IV., testing the strength of a solid paraffin of melting-point 76°–80° C., the values for the equivalent thickness of air and corresponding difference of potential are also evidently too low, as in other tests, at different points of the cake, the same pieces would not break down under the greatest electromotive force that could be obtained by the machine. In the measurements of the strength of solid paraffin of melting-points 42°–44° C., and 46°–48° C., Tables V. and VI., some of the results may have been vitiated by the balls pressing into the soft wax. I attempted to eliminate these errors by measuring the thickness of the cake after the disruption and at the point punctured.

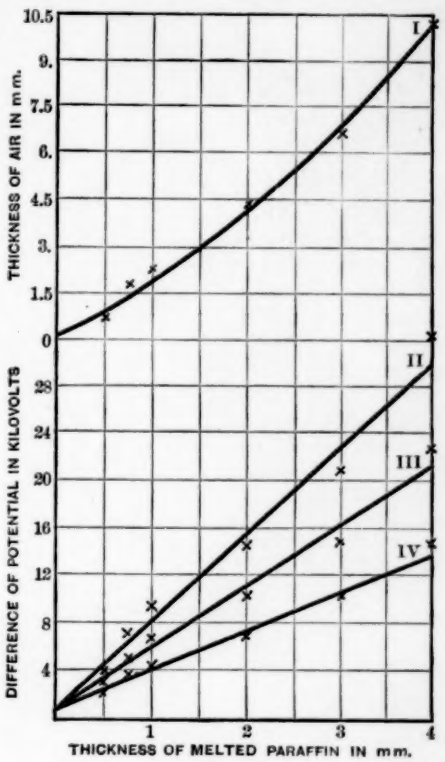

Fig. 2.

Though the results obtained with solids for the electrostatic gradient do not agree with one another as well as might be desired, still there is nothing to disprove the supposition that it is constant, and the more carefully the experiments were performed, the more nearly this quantity approached constancy.

Results.

FUNCTIONS FOR V (KILOVOLTS), r BEING MILLIMETERS.

Dielectric.	Own observations. Air reduced to P. D. by				Macfarlane. (*Trans. R. S. E.* Vol. 28.)	Steinmetz. (*Trans. A. I. E. E.* Vol. 10, No. 2.)
	Macfarlane's values.	Steinmetz's values.	Steinmetz's values modified as suggested at p. 104 below.	Baille's values.		
Paraffined paper	$36.0\,r$	$26.8\,r$	$60.7\,r$	—	—	$33.9\,r$
Beeswaxed paper	$55.2\,r$	$36.4\,r$	$82.3\,r$	—	—	—
Kerosene oil	$4.5\,r$	$3.4\,r$	$77.2\,r$	—	—	—
Melted paraffin. M. P. 52°	$5.5\,r$	$3.7\,r$	$8.4\,r$	$7.4\,r$	—	—
" " M. P. 76–80	$5.9\,r$	$4.0\,r$	$9.1\,r$	$8.1\,r$	—	$8.1\,r$
Solid paraffin. M. P. 52° .	$18.6\,r$	$12.4\,r$	$28.1\,r$	$24.6\,r$	$13.0\,r$	—
" " M. P. 42–44	$30.0\,r$	$18.6\,r$	$42.2\,r$	$42.0\,r$	—	—
" " 76–80 . .	—	$23.3\,r$	$52.9\,r$	$53.0\,r$	—	—
" " 46–48 . .	$30.0\,r$	$15.2\,r$	$34.5\,r$	$43.5\,r$	—	—
Turpentine	—	—	—	—	$9.4\,r$	$6.4\,r$

Discharge along the Surface of a Solid Dielectric.

In my experiments with melted paraffin, I observed that the plates, dipping down into the vessel containing the paraffin, were attracted towards one another, making the real distance between the plates less than the reading on the scale. To overcome this difficulty, Mr. Steinmetz separated his plates by pieces of mica or glass of measured thickness. In this way he kept the disks at constant distance, but probably introduced an error; for a break in the continuity of a dielectric weakens it, as I have shown by comparing the length of a spark in air with the length of one produced by the same electromotive force along the surface between air and glass. I had observed in the first experiments with solid dielectrics that the spark would pass much farther along the surface of paraffin than through an uninterrupted air gap. At first I attributed this phenomenon to the presence of particles of

conducting dust along the surface of the dielectric. On the 25th of April, 1894, I examined this subject more carefully, made a series of measurements of the surface lengths and equivalent air gaps, and obtained the results given in Table VII. and Fig. 5. Pieces of plate glass, broken so as to expose a smooth edge, were placed flat between one pair of the brass disks, with the smooth edge of the glass about the center of the disks; the machine was put in operation, and the other pair of disks were gradually separated until the spark preferred to pass along the surface of the glass. The distances between the pairs of plates were measured. I was greatly surprised to find that the ratio of the length of the spark in free air to that of the spark along the surface was very approximately constant, as expressed by the equation

$r = .442\, s$; where

r = sparking distance in air,

s = sparking distance along the surface between air and glass.

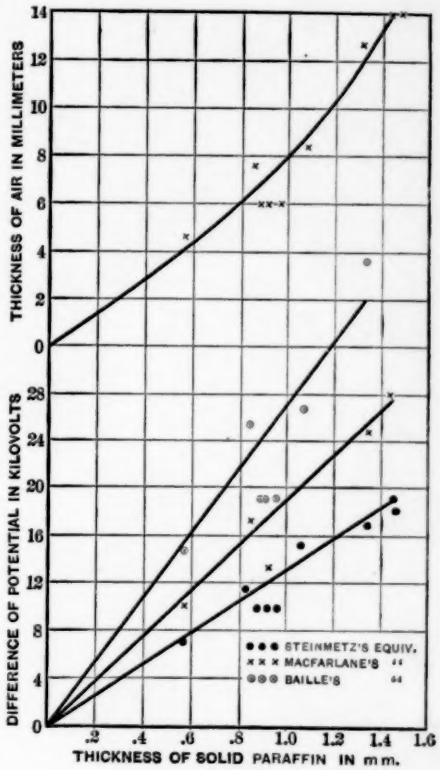

Fig. 3.

As all dust and moisture had been previously removed from the pieces of glass by rubbing them with chamois skin, this weakening of a gaseous dielectric by placing along with it a stronger dielectric could not have been due to the introduction of conducting particles. I have not had an opportunity to investigate the discharge along the boundary of other dielectrics, but believe that such an investigation would throw considerable light on the anomalous behavior of gaseous media.

Theoretical Examination of Results.

My experiments have not been conclusive. Not having at command the means of making absolute measurements of E. M. F., I have been compelled to make use of the air equivalents obtained by other experimenters. Their results do not agree. Dr. Macfarlane informs me that, while he is certain that his values are relatively correct, having carefully reduced them all to the same scale, he is not certain as to the correctness of the absolute magnitude. Mr. Steinmetz, in determining the electric strength of air, separated his plates by pieces of glass and mica, as he had done in his experiments on liquids, thus causing the spark to pass at much smaller P. D. than when the air between the plates was uninterrupted. He obtained the equation

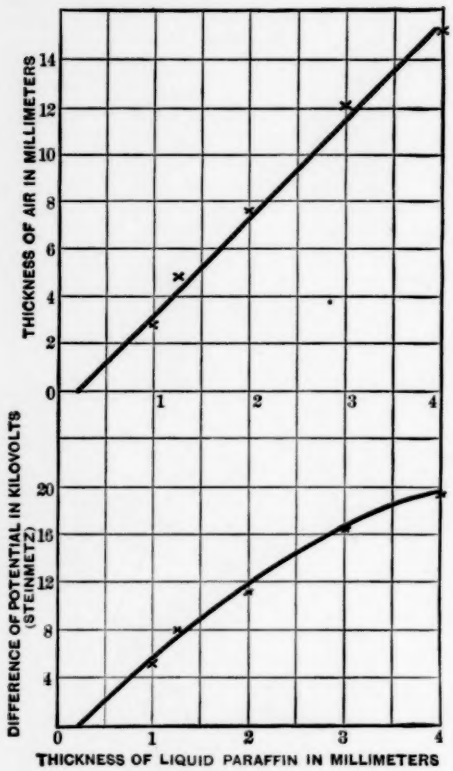

Fig. 4.

$$r = \underset{\text{milli-cent}}{36(e^{-1.3V} - 1)} + \underset{\text{kilovolts}}{54\,V} + 1.2\,V^2.$$

According to my measurements above, Table VII. and Fig. 5, this value of r should be multiplied by .442, which would give, dropping the exponential term as it vanishes for all but very low values of r,

$$r = 24.268\,V + .5304\,V^2.$$

Steinmetz's measurements, thus modified, agree very well with those made by other experimenters, as may be seen by referring

to Table IX. This table contains only observations of the length of sparks through air between parallel metal plates, produced by the given difference of potential.

The values of Mr. Steinmetz and Dr. Macfarlane are both probably correct relatively. The values given by Baille, in *Annales de Chimie et de Physique*, [5] 25, p. 486, 1882, appear to be the most reliable as to absolute measurement. Notwithstanding these discrepancies, I think we may consider it established that for solids and liquids the difference of potential necessary to force a spark through a dielectric is proportional to the length of the spark, or the electric strength is constant. This may be expressed by an equation of the form $V=kr$, in which V is the electromotive force, r the length of the spark, and k a constant depending on the substance tested.

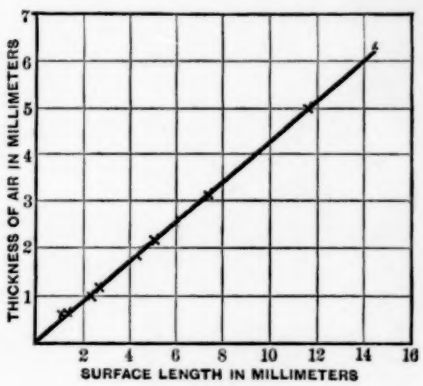

Fig. 5.

For gaseous dielectrics, the electrostatic gradient diminishes as the distance between the plates is made greater, or the dielectric weakens as the length of the spark increases. In this case the potential difference can be expressed as a function of the thickness of the dielectric only by an equation of a higher degree.

Upon examination of results, it appeared that this difference of behavior is due to convection in the case of the gas. This belief was strengthened by observing that when particles of dust or other solid impurities were suspended in a liquid, and the current passed as before, the liquid became very much agitated. The little particles were repelled from both plates, met in the middle with such energy as to heap up well-defined ridges at the surface, and then seemed to return to the plates as if they were alternately charged and discharged, as is the case with the air or dust particles in convection. When the liquid behaved in this way, I obtained a curve of results that, instead of being straight, was concave to

the axis of abscissæ like the air curve. See Table VIII. and Fig. 4.

This assumption of convection seems to give the equations required.

First let us consider pure disruption. The energy of the condenser manifests itself as stress on the intervening insulating medium. At the moment of disruption, the total energy being dissipated, we have

$$W=\frac{AK}{8\pi r}V^2=Arf,$$

in which A=area of one of the plates, r=the distance between the plates, and f=the quantity of energy per unit of volume required to disrupt the dielectric. Whence

$$V^2=\frac{8\pi f}{K}r^2, \quad V=\sqrt{\frac{8\pi f}{K}}r.$$

In pure disruption f is constant and proportional to the resistance to the passage of electricity from each molecule to the next adjacent molecule.

When there is convection, a part of the energy is used up in charging the layers of molecules of the insulator in contact with the plates.

(The subsequent motion of the particles across the intervening space is a remanifestation of the energy expended in charging them, and does not require consideration.)

The molecules along the surface being dielectric, have a certain repugnance for electricity. The total surface resistance will be proportional to the area of the surface and repugnance per unit of area, as expressed by the equation $R=Ap$.

In discharge by both disruption and convection, as in air and other gases, this quantity must be added to Arf above, and equated to the total energy; whence,

$$\frac{AK}{8\pi r}V^2=Arf+Ap,$$

$$V^2=\frac{8\pi f}{K}r^2+\frac{8\pi}{K}rp,$$

$$V=\sqrt{\frac{8\pi}{K}}\sqrt{fr^2+pr}.$$

This formula agrees with that for air obtained experimentally by Dr. Macfarlane while at Edinburgh; namely,

$$V = 66.94\sqrt{r^2 + .20503\, r},$$

and seems to be the rational expression of V in terms of r, if the discharge is purely disruption and convection. It is possible that, when the plates are exceedingly near each other, the condensation of the gas at the surface may introduce other terms. These approach zero as the distance increases, and may be neglected except for very small values of r. The formulæ deduced above apply only to discharge between plates of very great area compared with the distance between them.

PHYSICAL LABORATORY, UNIVERSITY OF TEXAS.

TABLE I.

MELTED PARAFFIN. MELTING-POINT 52° C.

Thickness of paraffin in millimeters.	Thickness of air in millimeters.						Dif. of potential in kilovolts. (Average.) (Macfarlane.)	Dif. of potential in kilovolts. (Average.) (Steinmetz.)	Dif. of potential in kilovolts. (Average.) (Baille.)
	Temp. 83°–85° C.	Temp. 75°–80° C.	Temp. 70°–73° C.	Temp. 60°–65° C.	Temp. 55°–58° C.	Average.			
0.5	—	0.80	1.0	1.0	0.80	0.90	3.30	2.2	4.0
1.0	2.2	1.90	1.9	2.1	2.15	2.05	6.00	4.0	7.7
1.5	3.0	2.70	2.8	3.0	3.50	3.06	8.10	5.3	10.6
2.0	4.1	3.70	4.0	4.1	4.60	4.12	10.20	7.0	13.8
2.5	5.7	5.00	5.7	5.3	5.20	5.40	12.95	8.9	17.6
3.0	7.3	6.70	—	7.2	—	7.07	15.60	10.9	22.5
3.5	8.8	8.35	—	8.4	—	8.50	18.90	12.3	27.0

January 24, 1894.

TABLE II.

MELTED PARAFFIN. MELTING-POINT 76°–80° C. TEMPERATURE 80° C.

Thickness of paraffin in millimeters.	Thickness of air in millimeters.	P.D. in kilovolts. (Steinmetz.)	P.D. in kilovolts. (Macfarlane.)	P.D. in kilovolts. (Baille.)
0.5	0.90	2.2	3.3	4.0
0.75	1.90	3.6	4.8	7.4
1.00	2.25	4.1	6.6	8.4
2.00	4.25	7.0	10.2	14.2
3.00	6.50	10.2	15.0	20.6
4.00	10.25	14.7	22.5	32.1

TABLE III.

SOLID PARAFFIN. MELTING-POINT 52° C.

Thickness of paraffin in millimeters.	Thickness of air in millimeters.	P. D. in kilovolts. (Steinmetz.)	P. D. in kilovolts. (Macfarlane.)	P. D. in kilovolts. (Baille.)	Gradient kv. per mm. (Steinmetz.)	Gradient kv. per mm. (Macfarlane.)	Gradient kv. per mm. (Baille.)
0.92	6.0	9.7	13.5	19.2	10.5	14.6	20.9
1.32	12.6	17.0	25.2	39.7	12.9	19.1	30.0
0.96	6.0	9.7	—	19.2	10.1	—	20.0
1.44	14.0	18.5	—	—	12.8	—	—
0.58	4.6	7.5	10.8	15.2	12.9	18.6	26.2
1.42	14.0	18.5	28.2	—	13.0	19.9	—
0.91	6.0	9.7	—	19.2	10.7	—	20.1
0.82	7.8	11.5	17.1	24.9	14.0	20.8	30.3
1.06	8.3	15.2	—	26.3	14.3	—	24.8
	Average gradient				12.4	18.6	24.6

TABLE IV.

SOLID PARAFFIN. MELTING-POINT 76°–80° C. BALL ELECTRODES.

Thickness of paraffin in millimeters.	Thickness of air in millimeters.	P.D. in kilovolts. (Steinmetz.)	P.D. in kilovolts. (Baille.)	Gradient kv. per mm. (Steinmetz.)	Gradient kv. per mm. (Baille.)
1.00	13.7	18.2	—	18.2	—
0.72	13.5	18.1	—	25.0	—
0.535	10.5	15.0	33.0	28.0	61.7
0.915	11.5	16.0	38.0	17.5	41.5
0.495	9.4	13.7	29.7	27.7	55.9

April 23, 1894.

Remark. — The above values for the thickness of air and difference of potential are evidently very low, as in other tests the same paraffin cakes would not break down under a much greater difference of potential.

Table V.

SOLID PARAFFIN. MELTING-POINT 42°-44° C. AIR ELECTRODES, PLATES; PARAFFIN ELECTRODES, BALLS.

Thickness of Paraffin in millimeters.	Thickness of air in millimeters.						Difference of potential in kv.			Gradient kv. per mm.		
	1st Trial.	2d Trial.	3d Trial.	4th Trial.	5th Trial.	Mean.	Macfarlane.	Steinmetz.	Baille.	Macfarlane.	Steinmetz.	Baille.
0.46	6.0	6.6	7.1	5.6	6.3	6.32	14.4	10.0	20.0	31.3	21.4	43.7
1.08	12.8	10.1	11.8	11.2	11.4	11.46	—	16.0	—	—	14.8	—
0.47	5.2	7.4 6.0	5.4	5.2	5.2	5.73	13.5	9.2	18.5	28.8	19.6	40.0
1.65	24.6	24.8	—	—	—	24.70	—	—	—	—	—	—
	Average gradient									30.0	18.6	42.0

Table VI.

SOLID PARAFFIN. MELTING-POINT 46°-48° C. AIR ELECTRODES, PLATES; PARAFFIN ELECTRODES, BALLS. PARAFFIN SOFT.

Thickness of paraffin in millimeters.	Thickness of air in millimeters.						P. D. in kilovolts.			Gradient kv. per mm.		
	1st Trial.	2d Trial.	3d Trial.	4th Trial.	5th Trial.	Average.	Steinmetz.	Macfarlane.	Baille.	Steinmetz.	Macfarlane.	Baille.
0.56	6.2	6.6	5.0	6.2	6.0	6.00	9.5	13.5	19.0	17.0	24.11	34.0
0.87	8.4	7.8	8.4	8.5	7.1	8.00	12.0	17.4	25.5	23.0	20.00	29.3
1.04	11.8	10.5	10.6	12.7	11.2	11.27	15.5	—	—	14.9	—	—
0.45	8.7	7.4	6.7	7.9	—	7.70	11.7	17.1	24.6	26.0	38.00	54.6
1.28	13.8	11.3	13.5	12.2	—	12.70	17.2	—	—	13.4	—	—
0.28	4.8	4.7	4.5	4.0	4.7	4.54	7.7	10.5	15.5	27.5	37.50	55.5
0.66	9.6	9.6	9.6	9.4	9.0	9.44	13.5	20.8	28.7	20.3	31.50	43.4
1.57	12.5	11.8	11.0	—	—	11.80	16.2	—	—	10.3	—	—
	Average gradient									15.2	30.00	43.5

April 27, 1894.

TABLE VII.

DISCHARGE ALONG THE SURFACE OF GLASS COMPARED WITH AIR.

s. Length of surface of glass in millimeters.	r. Thickness of air in millimeters.	Specific resistance. r/s.	Remarks. Average of
1.40	0.65	0.464	2 readings.
1.30	0.64	0.492	2 "
2.27	1.00	0.441	2 "
2.42	1.05	0.434	2 "
4.86	2.10	0.432	2 "
4.26	1.80	0.422	2 "
7.19	3.10	0.431	2 "
11.74	5.00	0.426	2 "
14.10	6.20	0.440	2 "

Mean specific resistance, 0.442.

$r = 0.442\ s$;

Where r = striking distance in air,

s = striking distance between glass and air.

April 25, 1894.

TABLE VIII.

CONVECTION EFFECT. MELTED PARAFFIN. MELTING-POINT 52° C. CONTAINING DUST PARTICLES PRODUCING PERCEPTIBLE CONVECTION. TEMPERATURE 70°–65° C.

Thickness of liquids in millimeters.	Thickness of air in millimeters.	Difference of potential in kilovolts. (Macfarlane.)	Difference of potential in kilovolts. (Steinmetz.)
1.00	3.0	7.01	5.4
1.25	4.8	11.40	8.0
2.00	7.6	16.65	11.5
3.00	12.1	—	16.5
4.00	15.0	—	19.6

TABLE IX.

DIELECTRIC AIR. LENGTH OF SPARK IN MILLI-CENTIMETERS.

Difference of potential in kilovolts.	Steinmetz — Disks 5 cm. diameter. (*Trans. Am. Inst. El. Eng.* February, 1893.)	Steinmetz's observations multiplied by 0.442.	Baille—plates. (*Thomson's Recent Researches in Electricity and Magnetism*, p. 70.)	Alex. Siemans — Disks of 10 and 3.7 cm. diameters. (*Elektrotechnische Zeitschrift*, 1892, No. 19, p. 246.)	Bourne — Plates of 6.3 cm. diameter. (*Electrical Engineer*, N.Y., 1892, No. 205, p. 361.)	De la Rue and Müller. (*Mascart and Joubert*, Vol. 2, p. 195.)	Sir Wm. Thompson — Disks. (*Mascart and Joubert*, Vol. 2, p. 197.)	Macfarlane — Disks of 10 cm. diameter. (*Trans. Roy. Soc. Edinbr.*, 1878, Vol. 28, p. 633.)
1	30	13.3	12.5	24	—	21	13	—
2	77	31	35	48	—	43	42	—
3	132	58	60	73	—	66	62	—
4	192	85	90	105	—	91	100	112
5	255	113	120	139	150	118	127	135
6	323	143	150	171	—	147	—	160
7	396	174	177	206	—	180	—	172
8	468	207	210	240	—	215	—	213
9	540	239	245	277	—	250	—	260
10	622	275	280	317	300	286	—	305
12	800	354	350	400	—	365	—	405
14	988	437	420	480	—	—	—	505
15	—	—	450	—	570	—	—	—
16	1173	518	485	598	—	—	—	605
18	1320	583	555	755	—	—	—	705
20	1468	649	630	940	1000	—	—	800
25	2200	972	—	—	1300	—	—	—

A GENERAL THEORY OF THE GLOW-LAMP.[1] I.

BY H. S. WEBER.

THE phenomena of the glow-lamp are perhaps the simplest in the domain of radiation. The lamp consists of a conductor in a nearly complete vacuum, traversed by an electric current under the action of which it is heated to a stationary temperature such that the radiation from its surface consists in considerable part of visible rays. The simplicity of the conditions lead to the supposition that between the size and the temperature of the radiating surface, the wave-length and intensity of radiation, and the quantities by means of which the electrical energy developed in the filament are determined, there exist relations which are capable of expression in some simple form.

Up to the present time, only purely empirical attempts have been made to determine the relation between the amount of light emitted by the glow-lamp and its electrical constants. Jamieson and Preece believed the brightness of the glow-lamp to be proportional to the sixth power of the strength of the current. Voit concluded from the observations of Jamieson, and from numerous measurements made at the electrical exhibition of Munich upon many varieties of lamps, that the light was approximately proportional to the third power of the energy expended. Hess has shown that whenever the range of intensities becomes considerable, a member must be added to Voit's expression which is proportional to the energy. Finally, Slotte has drawn the conclusion from simultaneous measurements of the strength of the current and the intensity, that the light of the lamp is closely related to the fourth power of the current strength.

These experimental investigations could lead to no final result,

[1] A paper read at the Frankfort Congress for Electrotechnics. Translated for the Physical Review, by E. L. N.

since the most important element, viz. the temperature of the glowing filament, was left altogether out of account. Finding that the means of determining this temperature were lacking, only the most unreliable estimates concerning it, running from 2250° to 1200°, being attainable, I have spent several years in an attempt to establish an exact physical theory of the electrical glow-lamp which should cover all the observed facts, and have for this purpose made exact measurements upon more than thirty types of lamp, in order to determine the characteristic properties and to obtain the values of such constants of radiation for carbon as are necessary to a completion of the theory. I believe that I have now reached a point in these attempts where all the phenomena of the glow-lamp may be expressed by means of a few formulæ. Permit me to report briefly in the following pages the results of my investigations.

I.

The starting-point of my work was the establishment of a general expression by means of which the intensity of homogeneous radiation of a given wave-length emitted by a solid body at a given temperature might be expressed in terms of the wave-length and temperature, and of the size and nature of the radiating surface. No such general formula existed in the realm of radiation.

In 1888 I showed that the result of a great variety of observations could be expressed in this form, —

$$d^2S = \frac{dF \cdot dF_1 \cdot \cos w \cdot \cos w_1}{r^2} \cdot \frac{c}{\lambda^2} \cdot e^{\alpha T - \frac{\beta}{\lambda^2 T^2}}, \qquad (1)$$

in which dF is an element of surface of the radiating body, dF_1 an element receiving this radiation, r the distance between the two elements of surface, w and w_1 the angles between the elements of these normals and the direction of the line joining them (r). This equation expresses the amount of energy of homogeneous radiation of the wave-length which is sent out from the surfaces of the element dF at the absolute temperature T to the surface of the element dF_1 in the unit of time. In this equation e has the usual

signification; a is a constant common to all solids, viz. the value 0.0043; b^2 is a constant which should have the same, or nearly the same, value for various bodies; and, finally, c is a constant which varies from substance to substance. To distinguish them, I have named these three, respectively, constants of temperature (a), the illuminating power (b^2), and the constant of emissivity (c).

From Equation 1 it follows, in the first place, that the amount of energy radiated from the entire surface F in all directions in the unit of time is

$$s = c \cdot \pi \cdot F \cdot e^{aT - \frac{1}{b^2 \lambda^5 T^2}}. \qquad (2)$$

This quantity is to be called the intensity of homogeneous radiation.

It follows, further, that the sum of the intensity of all the homogeneous rays which are emitted at the temperature T (the total radiation of the body) is expressed by the equation

$$S = \int_{\lambda=0}^{\lambda=\infty} s \cdot d\lambda = c \cdot \pi \cdot F \cdot \frac{\sqrt{\pi}}{2} \cdot b \cdot T \cdot e^{aT} = C \cdot F \cdot T \cdot e^{aT}. \qquad (3)$$

The quantity $C = \frac{1}{2}\pi\sqrt{\pi c \cdot b}$ is to be called the constant of total radiation.

If the radiating body k is placed in an enclosed space, the walls of which have the common temperature T_0, the total loss of energy which K suffers in consequence of the radiation in the unit of time is given by the formula

$$\Delta S_{T,T_0} = C \cdot F(T \cdot e^{aT} - T \cdot e^{aT_0}), \qquad (4)$$

it being assumed that the surface F of the radiating body is very small as compared with the surfaces of the surrounding walls, an assumption which is admissible for all incandescent lamps. I have assured myself that Equation 2 expresses the result of all trustworthy measurements hitherto made upon the relation between wave-length and temperature of homogeneous rays. And it can be shown also that Equation 4 agrees completely with the carefully determined results of Schleiermacher upon the total radiation of

platinum and the oxide of copper for temperatures between 273° and 1173°. Measurements of my own upon the total radiation of carbon between 300° and 700° were also capable of being expressed by Formula 4.

II.

Equation 4 affords a simple method for the determination of the temperature of the glow-lamp. After the temperature of the filament under the action of the current has become stationary, the amount of energy developed by the current in each unit of time plus the energy which the filament receives from the surrounding walls is equal to the energy which it gives out, so that for every such time-interval the equation

$$\Delta S_{T,T_0} = C \cdot F(T \cdot e^{aT} - T_0 \cdot e^{aT_0}) = i \cdot \Delta P$$

holds true, where i and ΔP are the current intensity which is regarded as constant, and the difference of potential between the terminals of the filament, respectively.

In the case here assumed, where $\Delta S_{T,T_0}$ is given in heat units, the equation takes the following form:

$$\Delta S_{T,T_0} = C \cdot F(T \cdot e^{aT} - T_0 \cdot e^{aT_0}) = \frac{i \cdot \Delta P}{J},$$

in which J is the mechanical equivalent of the calorie. It follows from this equation that by means of measurements of the quantities i, ΔP, T and F, the temperature of the filament can be determined, provided the constant C is known. This can be found by the estimation of the stationary temperatures which the filament attains under the action of known currents and potential differences. The value of these temperatures is given from the resistance of the filament, provided the dependence of resistance upon temperature has been determined by a series of experiments for the intervals of temperatures not given. The following table gives an example of the determination of the constant C, the total radiation of the filament of a specimen of the new Cruto lamp, the surface of which had been found by the length of the carbon and its cross-section at eleven points distributed along the filament

at equal distances. This surface was found to be 0.647 sq. cm. The resulting value of $C=0.0000171$ refers to gram-calories, centimeters, and seconds.

NEW CRUTO LAMP.

$F = 0.647$ sq. cm.

T	W	T	W
290°.0	332.08 Ω	440°.4	318.09 Ω
340°.3	327.68	490°.0	313.03
390°.3	323.01	540°.5	307.88

T_0 in the beginning = 290°.0; at the end = 290°.1.

i	ΔP	W	E	$\Delta S_{T,T_0}$	T	$\frac{T}{T_0}e^{a(T-T_0)}-1$	C
0.006526 A.	2.1472 V.	329.02 Ω	0.01401 W.	0.003344 cal.	325°.0	0.3027	169.2×10^{-7}
0.010051	3.2729	225.62	0.03289	0.007850	362°.5	0.7096	171.0
0.13176	4.2449	322.17	0.05593	0.013349	398°.9	1.1971	172.4
0.16542	5.2655	318.31	0.08710	0.020740	438°.2	1.8577	171.0
0.19986	6.2863	314.53	0.12564	0.029983	475°.3	2.6399	173.9
0.23358	7.2637	310.97	0.16966	0.040451	510°.2	3.5315	175.4
0.26690	8.2022	307.31	0.21892	0.052248	546°.1	4.6677	171.4

$M = 171.8\times10^{-7}$.

Following this plan, I have in the course of years determined the constant of total radiation for thirty-three varieties of glow-lamp. The summary of the results obtained shows that these thirty-three filaments belong to two groups. Three carbons, that of the Edison lamp, the new Cruto lamp, and of the lamp of Woodhouse and Rawson, have a radiation coefficient of 0.0000169 and 0.0000174; while thirty filaments give for the radiation-constant values between 0.0000127 and 0.0000132. The three carbons first-named are black; the others show gray surfaces. The mean value of C for these two different varieties of filament stand in the relation of 100 to 75.5. This is almost exactly the same relation

which Leslie, at the beginning of this century, found for lamp-black and graphite; viz. 100 to 75. It may, therefore, be asserted that incandescent lamp filaments with gray surfaces should be classified as graphitic carbon.

A knowledge of the constant C makes it possible to determine the temperature of the filament for every condition of incandescence when we know the size of its radiating surface. It is interesting to know what the temperature of the filament is when the incandescent lamp is in operation at normal brightness. The result is a surprising one; viz. that the normal temperature of all sorts of lamps is very nearly the same, and is included in an interval between $T=1565°$ and $T=1580°$ of the absolute scale. In the case of some lamps of very great brilliancy — that is to say, of lamps with thick filaments, which can be brought to higher temperatures without serious damage, and which can consequently be operated at higher economy — we find the normal temperature to be about 40° higher.

It is, further, of importance to know between what temperatures the filament changes when the candle-power of the lamp is varied through considerable range: from 2 candles to 30 candles for a 16 candle-power lamp, for example, or from 20 to 300 candles in the case of a 200 candle-power lamp. The result of measurements shows that the temperature variations which correspond to such changes in brightness amount to about 180°. In practice, accordingly, glow-lamp illumination covers a range extending from 1400° to 1600° for small lamps, and 1450° to 1650° for large lamps. If we confine ourselves in the study of incandescent lamps to this range of temperatures, we may write, in place of Equation 5, the simple form —

$$\Delta S_{T,T_0}=C\cdot F\cdot T\cdot e^{aT}=\frac{i\Delta P}{J}, \qquad (6)$$

since in these cases the quantity $T_0\cdot e^{aT}$ is very small as compared with the quantity $T\cdot e^{aT}$. This is shown by the following small table: —

T_0	$T_0 \cdot e^{aT_0}$	T	$T \cdot e^{aT}$
273 + 50°	1.295×10^8	1400°	576×10^8
+ 75°	1.554	1450°	740
+ 100°	1.854	1600°	1554
+ 125°	2.203	1650°	1989

In this simpler form, Equation 6 indicates in what degree the current must increase in order to bring about a given rise of temperature in the filament. To make this matter clearer, I have drawn up the following table:—

T	$T \cdot e^{aT}$	T	$T \cdot e^{aT}$	T	$T \cdot e^{aT}$
1400°	576×10^8	1500°	948×10^8	1600°	1554×10^8
1410	606	1510	996	1610	1634
1420	637	1520	1047	1620	1717
1430	670	1530	1100	1630	1803
1440	704	1540	1156	1640	1894
1450	740	1550	1214	1650	1989
1460	777	1560	1276	1700	2541
1470	816	1570	1340	1800	4136
1480	858	1580	1408	1900	6712
1490	902	1590	1479	2000	10860

From these data, and from the expression

$$\frac{d(T \cdot e^{aT})}{dT} \cdot \frac{1}{T \cdot e^{aT}} = \frac{1}{T} ta,$$

it may be seen that for the entire range of temperature from 1400° to 1600°, the following simple law holds true: viz. that every increase in the energy of the current amounting to $\frac{1}{2}$ per cent raises the stationary temperature of the incandescent lamp 1°.

To show in greater detail the changes which the filament suffers by change of current and potential difference, and the changes which the resistance of the filament and the amount of light emitted suffer through change of temperature, I introduce the four following tables, which may also serve to indicate the

manner in which the investigation of thirty-three types of incandescent lamps was carried out. In the eight vertical columns of the tables are given respectively ten values: The strength of the current i, the difference of potential ΔP, the resistance W, the energy E, the candle-power H, the quantity $\frac{H}{E_3}$, the efficiency $E_1 = \frac{E}{H}$, and the absolute temperature T. As unit of brightness, the British standard candle has been selected, and H denotes the mean horizontal intensity. Of the lamps to which these four tables refer, two had black carbons and two gray carbons. The quantity R in each table refers to the radiation factor for brightness—the factor, namely, by which the mean horizontal candle-power is to be multiplied in order to get the mean spherical candle-power. These quantities were determined by measuring the lamp in 132 positions as evenly distributed through space as possible.

WOODHOUSE AND RAWSON'S LAMP.

("60 V., 30 C.P.")

$F = 0.889$ sq. cm. $R = 0.780$. $C = 0.0000169$.

i	ΔP	W	E	H	$q = \frac{H}{E_3}$	$E_1 = \frac{E}{H}$	T
0.8768 A.	41.41 V.	47.23 Ω.	36.31 W.	1.82 N.K.	38.1×10^{-6}	19.95 W.	1400°
0.9855	45.18	45.85	44.52	3.60	40.8	12.37	1441
1.0967	49.01	44.69	53.74	6.55	42.2	8.20	1479
1.2108	52.79	43.60	63.90	11.07	42.4	5.77	1514
1.3264	56.72	42.76	75.23	17.85	41.9	4.21	1548
1.3853	58.60	42.30	81.18	22.38	41.8	3.63	1561
1.4460	60.59	41.90	87.61	27.61	41.1	3.17	1578
1.5058	62.48	41.49	94.08	34.13	40.9	2.76	1592
1.5675	64.46	41.12	101.05	41.16	39.9	2.55	1606
1.6290	66.32	40.71	108.63	48.35	38.4	2.23	1620

SUNBEAM LAMP.

("80 V., 5 A., 200 C. P.")

$F = 5.04$ sq. cm. $R = 0.711$. $C = 0.0000131$.

i	ΔP	W	E	H	$q = \frac{H}{E_s}$	$E_1 = \frac{E}{H}$	T
4.064 A.	53.90 V.	13.26 Ω.	219.0 W.	22.72 N.K.	2.16×10^{-6}	9.64 W.	1463°
4.218	55.74	13.21	235.1	28.47	2.19	8.26	1478
4.442	58.49	13.17	259.8	38.77	2.21	6.70	1498
4.747	62.26	13.12	295.5	56.82	2.20	5.20	1524
5.056	66.07	13.07	334.0	81.23	2.18	4.11	1549
5.377	70.00	13.02	376.4	115.1	2.16	3.27	1573
5.689	73.82	12.98	420.0	156.9	2.12	2.68	1595
5.980	77.49	12.96	463.4	205.8	2.07	2.25	1615
6.282	81.29	12.94	510.7	263.7	1.98	1.94	1636
6.584	84.99	12.91	559.6	334.7	1.89	1.69	1654

LAMP OF THE ALGEMEINEN ELEKTRIZITÄTS-GESELSHAFT, BERLIN.

("100 V., 16 C.P.")

$F = 0.760$ sq. cm. $R = 0.801$. $C = 0.0000129$.

i	ΔP	W	E	H	$q = \frac{H}{E_s}$	$E_1 = \frac{E}{H}$	T
0.4212 A.	77.19 V.	183.3 Ω.	32.51 W.	2.99 C.P.	87.2×10^{-6}	10.87 W.	1464°
0.4431	80.89	182.5	35.85	4.13	89.3	8.67	1483
0.4668	84.80	181.7	39.58	5.60	90.3	7.07	1503
0.4903	88.83	181.2	43.55	7.41	89.7	5.88	1522
0.5136	92.87	180.8	47.70	9.71	89.5	4.91	1541
0.5360	96.71	180.4	51.84	12.42	89.2	4.18	1557
0.5588	100.60	180.0	56.21	15.76	88.7	3.57	1574
0.5823	104.58	179.6	60.90	19.70	87.2	3.09	1591
0.6057	108.60	179.3	65.78	24.25	85.2	2.71	1607
0.6295	112.57	178.8	70.85	29.41	82.7	2.41	1621

NEW CRUTO LAMP.

("100 V., 0.59 A., 16 C.P.")

$F = 0.632$ sq. cm. $R = 0.734$. $C = 0.0000174$.

i	ΔP	W	E	H	$q = \frac{H}{E_3}$	$E_1 = \frac{E}{H}$	T
0.3948 A.	78.53 V.	198.9 Ω.	31.00 W.	2.21 C.P.	74.2×10^{-6}	14.03 W.	1434°
0.4233	83.40	196.9	35.30	3.32	75.9	10.63	1460
0.4506	87.56	194.3	39.45	4.69	76.4	8.41	1482
0.4924	93.79	190.5	46.18	7.58	77.0	6.08	1514
0.5180	97.60	188.4	50.56	9.93	76.8	5.09	1532
0.5444	101.12	185.7	55.05	12.75	76.4	4.32	1550
0.5706	104.41	182.9	59.58	16.10	76.1	3.70	1566
0.5990	108.27	180.4	64.85	20.67	75.8	3.14	1582
0.6305	112.11	178.7	70.68	26.60	75.3	2.66	1600
0.6579	115.99	176.3	76.32	33.06	74.2	2.31	1618

A RELIABLE METHOD OF RECORDING VARIABLE CURRENT CURVES.

BY ALBERT C. CREHORE.[1]

Introduction.

A PRACTICAL problem that has in more recent years presented itself to the electrician and physicist alike is "How shall we measure the exact current which flows in a conductor at any instant of time, and record all the irregular changes to which it is subject?" Probably every one who has thought of such matters at all has considered this problem in some of the phases which it presents. The importance of the question, since the introduction and extensive use of the alternating current, has emphasized the fact that we need a "reliable method" of measuring the instantaneous values of a variable current, which is not a "method by points," but "a method which continuously records the current."

Under "a method by points" is included any method in which the current is obtained from readings (usually of an electrostatic voltmeter) due to the charge of a condenser which may be connected in at any point of time. The essential characteristic of the method is that the current is *supposed* to repeat itself exactly during successive periods, or more generally when the conditions are exactly repeated. There can be no doubt that the current *does repeat* itself under exactly similar conditions, but can we be *sure* that those conditions *are exactly repeated?* By this method a number of points are found, the time occupied being at least several minutes, and the collection of points properly arranged is a representation of the current during as short a time as the one-hundredth of a second, perhaps. Yet this method has proved to

[1] A paper presented at the General Meeting of the American Institute of Electrical Engineers in Philadelphia.

be a very useful and practical one, and has given us information concerning the currents and potentials of generators and transformers which is of paramount importance. Yet all will agree that this "method by points" is too limited in its application, and does not show us any sudden temporary change taking place in a current which does not *repeat* itself. Such, for instance, as a sudden "make," or "break," or "change" in an alternating current would not be easily shown by this method. The second method, previously designated "*a method which continuously records the current*," is the one to which this paper more particularly refers. Under this head is included all methods which attempt to record the current by causing it, either directly or indirectly, to move a material something so that its displacement is some single valued function of the current. As an example of this method may be mentioned the well-known experiments of Froelich in which a telephone is used, upon the disk of which is mounted a mirror that permits a beam of light to be reflected from it. Any vibration of the disk gives an angular motion to the ray of light, and this motion is in turn recorded upon a moving photographic plate. Other examples might be mentioned in illustration of this method, for instance, a wire which is deflected in a magnetic field, or stream of mercury so influenced; but it will be noticed that in all of these cases an appreciable amount of *ponderable* matter is required to be moved backward and forward during each reversal of the current. When the current reverses hundreds of times per second, the unavoidable difficulty is introduced that the forced oscillations of this ponderable matter, no matter how small in amount, become so superimposed upon those of the current which it is desired to measure that they are inseparably mixed together; and the record does not show the true current, but the resultant vibrations of the instrument. That this is the case with the method of the telephone above referred to, has been established beyond a doubt it seems, by experiments conducted at Cornell University by Mr. Henry Floy. The current furnished to the telephone was carefully measured by the "method by points," and care was taken to see that the current as measured by points was the same as that used in the telephone. The vibrations of the telephone did not

even approximately agree with the current as measured by well-established methods.

Bearing these points in mind, and remembering the high frequency of some of the oscillations which it is desired to record, may we not with some degree of certainty predict that any of these methods requiring the rapid motion of ponderable matter will be open to precisely the same objections which are noticed in the case of the telephone? Without answering this question, probably all will agree that the difficulty may *certainly* be avoided by using as a vibrator, instead of this so-called "*ponderable matter*," a vibrator that has *no* weight. It is to this question of finding a form of vibrator *without weight* that I invite your attention.

The Weightless Vibrator.

The idea of the weightless vibrator is perhaps already suggested in the beam of light. But how shall we cause a beam of light to have a change in direction simply by means of a current flowing in a circuit without the intervention of some moving material? A way of influencing a beam of light directly by an electric current (or more properly by its magnetic field) is that discovered long ago by Faraday. It is by means of the discovery of the rotation of the plane of polarization by an electric current that I propose a method of obtaining a weightless vibrator. The explanation will be made clearer by reference to the diagram of apparatus (Fig. 1). A beam of light is passed through a polarizer (Nicol prism), so that the vibrations of the beam take place in only one plane upon emergence. If it is then passed directly through an analyzer (Nicol prism) the latter may be set at such an angle as to prevent all light from passing through it, and thus produce darkness beyond the analyzer. Faraday's discovery was that if a beam of polarized light is passed through some *substance* in the direction of the lines of magnetization within that substance, there is a rotation of the plane of polarization in a direction which is the same as the direction of the current required to produce such a magnetic field. The direction of rotation is unaltered, therefore, whether the light beam advances in the same or the opposite di-

rection to the magnetization, so that a beam reflected back and forth through the substance several times has its rotation increased by equal amounts each time. If the direction of the ray of light be at right angles to the lines of magnetization, there is no rotation produced. The amount of this rotation has been carefully investigated by Verdet, who announced laws by which it may be expressed. They are summed up in the following statement:

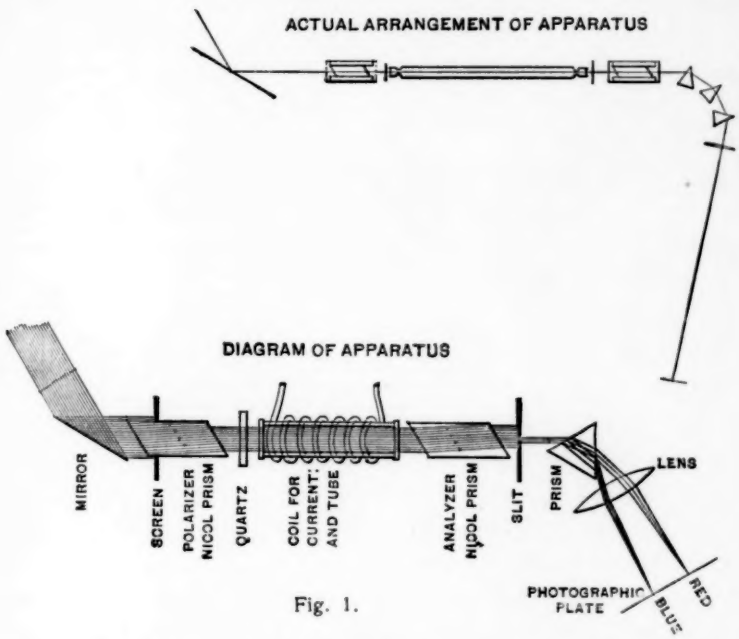

Fig. 1.

"The rotation of the plane of polarization for monochromatic light is in any given substance proportional to the difference in magnetic potential between the points of entrance and emergence of the ray"; that is, it is equal to a constant times this difference of potential, and is expressed by the formula

$$\theta = v V, \qquad (1)$$

where θ = angle of rotation, V = difference in magnetic potential, and v for a given wave-length is constant in any one substance. This constant is known as Verdet's constant. If now the light is passed through the polarizer and then through a tube containing the

substance used, around which is wound a coil of wire, and thence through the analyzer, an observer would find complete darkness upon looking through the analyzer when set in the crossed position. But if without moving the analyzer a current is sent through the coil on the tube, light appears to the observer. This is because the plane of polarization has been rotated by the current, and practically the prisms are no longer crossed. Now let the analyzer be rotated while the current is still flowing, and the observer will see a series of beautiful colors through the analyzer, a different one for each position of it; but as long as the current flows, he cannot produce darkness again by any amount of rotation of the analyzer.

This effect suggests what is known to be a fact, that the different wave-lengths composing white light are rotated by the current in different amounts, so that when the analyzer is turned to the angle corresponding to the yellow light, say, only the yellow light is prevented from passing through the analyzer. All the other rays, being rotated by different amounts, pass through the analyzer, and there being mixed together they give rise to the series of beautiful complex colors above mentioned. A different color is seen for each position of the analyzer because in each position a different color is subtracted from white light, and the observer sees what is left, or merely the complementary color.

The law which tells the amount of rotation given to the different colors is pretty accurately known; and theory in this case is in close accord with the observed facts. The equation which closely expresses the amount of the dispersion for the different wave-lengths may be written: —

$$v=\frac{cn^3}{\lambda^2}\left(1-\frac{\lambda}{n}\frac{dn}{d\lambda}\right), \qquad (2)$$

where v is the so-called Verdet's constant, λ the wave-length, and n the index of refraction of the medium. c is a constant for any one medium, which is, however, for different media, inversely proportional to the permeability of the medium. This is a formula at which Maxwell arrived from his theory of molecular vortices, and we shall see how closely it is in accord with observation. We see

by this formula that Verdet's constant depends not only upon the wave-length, but upon the index of refraction corresponding to that particular wave-length, and also upon the rate of change of the index with respect to the wave-length. If this rate of change of n with respect to λ is small, as would be the case with a substance where the dispersion is small, and the index of refraction regarded as approximately constant, then it is seen that the formula reduces to an extremely simple form, viz. : —

$$v=\frac{c_1}{\lambda^2} \tag{3}$$

Here Verdet's constant is inversely proportional to the square of the wave-length. Using this approximate form for the present, we see from Verdet's law, equation (1), that

$$\theta=v\,V=c_1\frac{V}{\lambda^2} \tag{4}$$

But the difference of magnetic potential, v, is $\frac{4\pi Si}{10}$, where Si is ampere turns, and thus we have

$$\theta=4\pi c_1 Si/10\,\lambda^2=c_2\,i/\lambda^2, \tag{5}$$

where

$$c_2=4\pi c_1 S/10. \tag{6}$$

A reference to Fig. 2 will show this relation between angle of rotation, wave-length, and current. Several spirals are shown, corresponding to the several lines of the spectrum, known as *A*, *B*, *D*, *F*, and *G*. The radii of the circles which intersect these spirals are proportional to the current flowing in the circuit, while the angle, which the radius, drawn to any point of intersection, makes with *OP*, represents the rotation for that particular wave-length and current. The spiral *OA* is in the extreme red, and *OG* in the violet of the spectrum, and the diagram thus indicates that the red rays are not rotated so much as the blue. The direction of rotation in the diagram is as indicated by the arrow. Now, returning to the observer looking through the analyzer, if he could resolve the light there seen into the pure colors of the spectrum, what he should expect would be, with no current, a complete spectrum,

since *all* rays are rotated by the current. But let him rotate the analyzer, and he finds that first one color then another disappears, and a *dark band* is seen to move across the spectrum as he rotates the analyzer. Again, let him rotate the analyzer to a certain angle and leave it there while he varies the current. He should expect that the band would move, but would vanish entirely with zero current, and thus prevent observation for small currents.

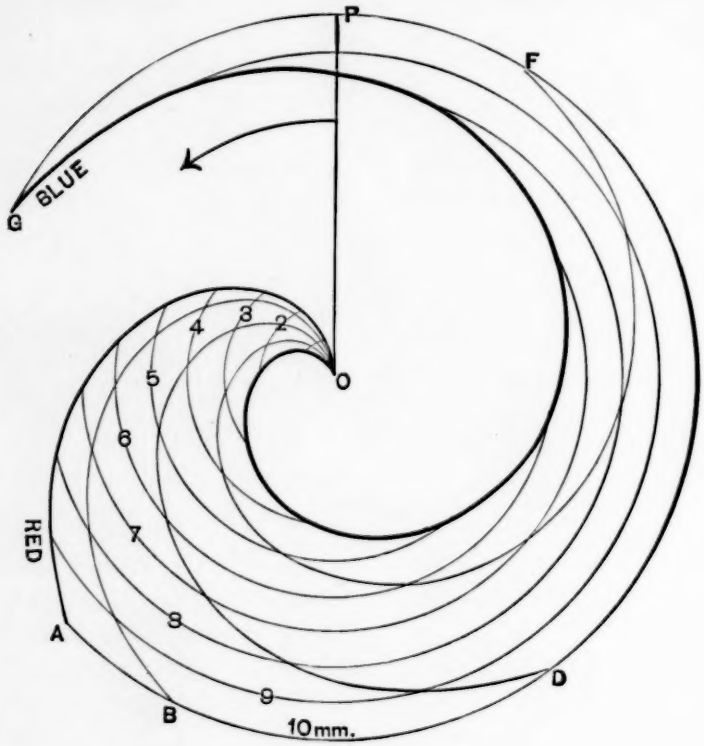

Fig. 2.

Fortunately we have substances which naturally rotate a beam of polarized light, for by means of this aid we may obviate the difficulty that the band vanishes with no current. For instance, a parallel plate cut from a crystal of quartz perpendicular to the optic axis has this property of rotating the plane of polarization. Quartz is selected for the material used because of its great trans-

parency and high specific rotary power. The law of the rotation is similar to that already mentioned for the rotation by the current. The approximate law is that the rotation is inversely as the square of the wave-length, which may be expressed: —

$$\phi = c_3 e/\lambda^2, \tag{7}$$

where ϕ is the angle of rotation for the wave-length λ, c_3 is a constant, and e is the thickness of the plate. The thickness of the quartz plate is seen to correspond to the current in equation (5). Fig. 2 represents the actual rotation for different thicknesses of quartz, each circle corresponding to a plate one millimeter thick. The equivalent of a quartz plate one millimeter thick is represented approximately by 35,700 ampere turns wound upon a tube containing carbon bisulphide. This latter is the substance used, being selected on account of its high transparency and specific rotation.

If a quartz plate be placed between polarizer and analyzer, the effect is the same as if the current circulated around the tube of carbon bisulphide, and we may, by rotating the analyzer, move the dark band completely across the spectrum by means of the quartz plate without any current. But suppose we set the analyzer so that the dark band remains in the center of the spectrum, and then pass a current through the coil. We observe a motion of this dark band back and forth through the spectrum as the current is repeatedly reversed. For any given current its position is always the same, so that its motion may be calibrated by passing known currents through the coil. Have we not in this found a weightless vibrator that is sure to move in unison with the currents, if the term is allowed?

Before passing on to the more practical side of the question, it may be asked, will this band move back and forth so that its displacement is approximately proportional to the current? The answer to this question lies so near at hand that your attention is invited to it for a moment. The rotation of the plane by quartz is approximately represented by the formula: —

$$\phi = c_3 e/\lambda^2.$$

The rotation by the current is represented by

$$\theta = c_3 i/\lambda^2.$$

If both these rotations take place together, the resultant is merely the sum of the components, and

$$\chi=\phi+\theta=(c_3e+c_2i)\frac{I}{\lambda^2}. \tag{8}$$

The position of the dark band depends upon the position of the analyzer. Let the analyzer be set at some convenient angle, a, with the position of complete darkness, and let it remain there. Then the wave-length or color where the dark band occurs for the quartz plate (being called λ_0) is given by (7) above, and we have

$$a=c_3\frac{e}{\lambda_0^2}. \tag{9}$$

The wave-length corresponding to this constant angle, a, when both quartz and current are used, is given by (8), and we have

$$a=(c_3e+c_2i)\frac{I}{\lambda_2}, \tag{10}$$

in which λ is that wave-length corresponding to a certain current, i, and therefore i and λ are coördinate variables. Equating the values of a, we have

$$\frac{c_3e}{\lambda_0^2}=\frac{c_3e+c_2i}{\lambda^2} \tag{11}$$

This may be written

$$\lambda^2=\frac{c_2\lambda_0^2}{c_3e}i+\lambda_0^2, \tag{12}$$

and in this form the relation between wave-length and current is seen to be represented by a parabola. In Fig. 3 are represented two sets of parabolas obtained from equation (12) by assuming that λ_0 and e take in succession different values. Where $e=2$ mm. the set of parabolas marked I. is obtained, and where $e=6$ mm. set II. is obtained. By giving λ_0 different values we merely vary the parameter of the parabola without changing the origin. It will be remembered that λ_0 represents that wave-length corresponding to the position of the dark band for no current. It is therefore the value of λ, when i is equal to zero, as appears from the equation independently. The axis of λ is the vertical line to the right of the figure, upon which the letters A, B, etc., are written. These letters

show the positions of the various Fraunhofer lines, and one parabola is drawn for each line. Each parabola then corresponds to one setting of the analyzer that the dark band is found at these lines of the spectrum for zero current. The upper parabola is at the red end, and the lower at the blue end, of the spectrum. The axis of current is the base line of the diagram, and currents to the left of the vertical line are called negative, while those to the right are called positive. The axes of all the parabolas coincide with

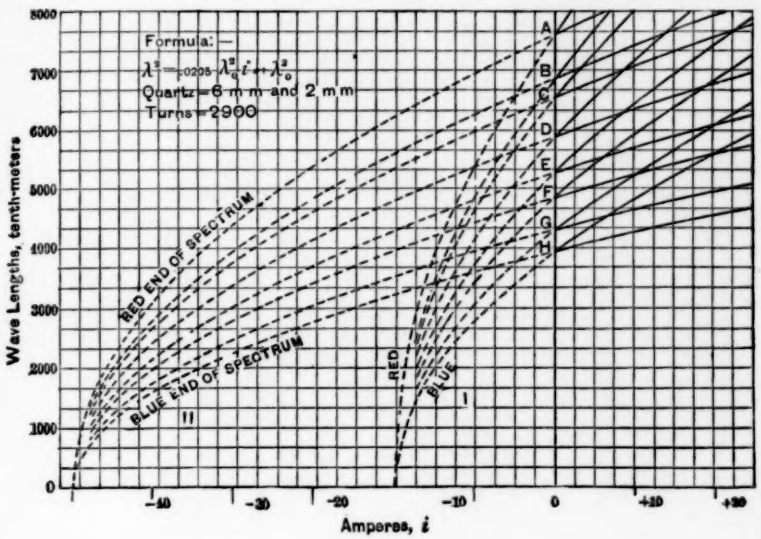

Fig. 3.

each other and with that of the current, and for a given quartz plate they all intersect this axis at the same point, so that taking different settings of the polarizer is equivalent to changing the parameter only of the parabola.

The interpretation of these results may be put as follows: If we have a spectrum in which the wave-lengths are proportional to the distances along the spectrum (which is the case with Professor Rowland's arrangement of a concave grating), then the displacement of the dark band to one side or the other, due to the current, will be exactly according to the shape of these parabolas near the zero point; that is, near the vertical line lettered *A*, *B*, etc. Since

it appears that each parabola at such a great distance from the origin is nearly a straight line, the displacement in such a spectrum will be nearly proportional to the current.

A noticeable feature, easily revealed by the graphical construction, is that in the red end of the spectrum, where the inclination of the parabola to the i axis is the greatest, the motion of the band will be the greatest for a given current.

Of course it is understood that this construction has to do with the relation between the wave-length and the current, and not between the displacement and current, unless the wave-length and displacement are proportional. It does not apply, for instance, to the displacement in the spectrum of most prisms. In the prisms used the red rays were so crowded together that the motion as observed was nearly the same in the red as in the blue. The width of the band, however, is for this reason narrower in the red than in the blue, — a consideration of considerable practical importance.

Description of Some of the Apparatus.

The tube upon which the coil carrying the current was wound was a glass tube 1.4 cm. internal, and 1.8 cm. external, diameter, and 70.15 cm. long. The tube was filled with carbon bisulphide, which was confined in the tube by means of two plane parallel plates of glass, each 1.3 cm. thick, fitted tightly upon the ground ends of the tube. Upon this tube were wound six layers of No. 18 double cotton copper magnet wire, occupying a length on the tube of 61.5 cm. The wire was wound so that 100 turns occupied 12.7 cm. Thus the total number of turns, 2900, is very large considering the size of wire.

The light used was sunlight reflected from the mirror of the heliostat.

The Nicol prisms are two fine specimens which were obtained by Dartmouth College at a time when larger specimens could be obtained than may now easily be found. The width of the slit does not need to be very narrow. A width of a quarter to a half millimeter will do better than a narrower one, because more light

is admitted to the photographic plate, and in passing through so many different substances even sunlight is rendered comparatively feeble by the time it strikes the photographic plate.

A further description of the apparatus is hardly deemed to be necessary, inasmuch as no claim is made to having obtained more than the most crude of first results, which may be the results obtained by apparatus arranged in a comparatively poor manner for the end sought. Yet the results obtained seem to be so promising for the future that the subject is presented to you at this early date in the experiment in the hope that it may soon receive an impetus from other experimenters who have better facilities than those at my disposal, and thus become a fruitful source of extending our knowledge of instantaneous current flow in conductors.

The objections which most naturally suggest themselves against this method of taking current curves are perhaps the following. The photographic plate must move so quickly that the time of exposure of any one part of the plate is extremely short. To meet this demand the most sensitive plates that can be made should be used. The width of the band with any given plate depends largely upon the time of exposure. Then, too, a plate is to be desired that will photograph toward the red end of the spectrum as well as in the blue. The band does not possess very sharp outlines, but gradually shades off from dark to light.

These objections do not have so much weight, however, in cases where the general direction of the variation of the current is what is wanted more than any exact measurement of its amount, and in the majority of cases this is really what is wanted. Yet it cannot be said that in these preliminary experiments the band used was nearly as sharp as may be obtained.

Another objection of a different nature that seems difficult to avoid is the fact that the coil, which is wound upon the tube, must necessarily possess a small amount of self-induction. It may be said, however, that even though we are prohibited from measuring certain currents on account of this self-induction, we are always sure that we are measuring the exact current which is flowing through the coil.

A More Exact Expression of the Relation between the Wave-Length and Verdet's Constant.

The approximate relation between the wave-length and Verdet's constant used above was that Verdet's constant varied inversely as the square of the wave-length. It is considered of sufficient interest to inquire just how nearly this is an approximate formula.

By reference to equation (2) it is evident that if we only knew the relation between the index of refraction and the wave-length, we might obtain the relation between the wave-length and Verdet's constant in terms of these two quantities alone and constants.

Such a relation is afforded by Briot's formula, which is a modification and improvement upon the well-known formula of Cauchy. This is

$$1/n^2 = k\lambda^2 + A + B/\lambda^2 + C/\lambda^4 + \cdots, \tag{14}$$

where n is the index of refraction corresponding to the wave-length λ, and k, A, B, etc., are constants for the given substance. Assuming that all terms beyond B/λ^2 are negligible, we may differentiate with respect to λ and obtain the equation

$$\frac{\lambda}{n}\frac{dn}{d\lambda} = n^2\left(\frac{B}{\lambda^2} - k\lambda^2\right). \tag{15}$$

Upon eliminating B/λ^2 between (14) and (15), we obtain

$$\frac{\lambda}{n}\frac{dn}{d\lambda} = 1 - 2kn^2\lambda^2 - An^2. \tag{16}$$

Substituting in (2) the expression thus obtained, we have

$$v = cn^5(2k + A/\lambda^2). \tag{17}$$

But by (14)

$$n^5 = (k\lambda^2 + A + B/\lambda^2)^{-\frac{5}{2}}. \tag{18}$$

Hence

$$v = c(2k + A/\lambda^2)(k\lambda^2 + A + B/\lambda^2)^{-\frac{5}{2}}. \tag{19}$$

This formula represents to a high degree of accuracy the observed values of Verdet's constant for carbon bisulphide. It probably would for any other substances, but carbon bisulphide is the only one to which it has been applied by me. The constants k, A, and B may be found by means of equation (14) and

the observed values of the refractive index for known wave-lengths. The values used are those observed by Messrs. Gladstone and Dale. (See Glazebrock's Physical Optics, page 243.)

Line of spectrum.	Index of refraction for carbon bisulphide.	Line of spectrum.	Index of refraction for carbon bisulphide.
A 7621	1.6142	E 5270	1.6465
B 6870	1.6207	F 4861	1.6584
C 6563	1.6240	G 4308	1.6836
D 5893	1.6333	H 3969	1.7090

These values give curve I. in Fig. 4. The lines *A*, *F*, and *H* were selected and three simultaneous equations formed from (14),

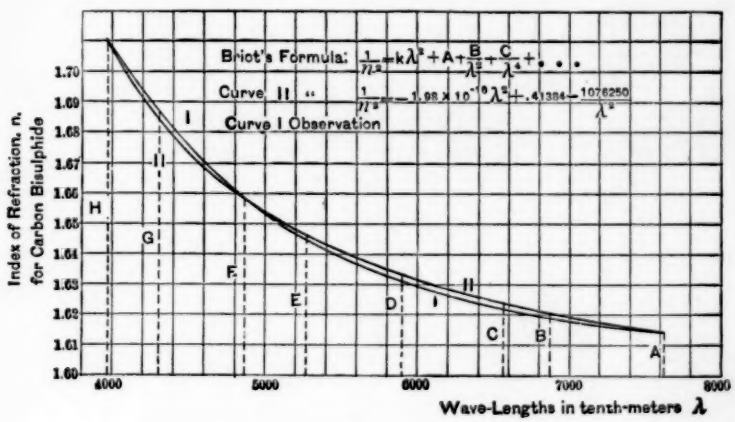

Fig. 4.

so that the resulting curve II. should pass through these three observed points.

The values

$$k = -1.98 \times 10^{-10}, \tag{20}$$

$$A = .41384, \tag{21}$$

$$B = 1076250., \tag{22}$$

were obtained by the determinant solution of these three simultaneous equations. The resulting black curve may not appear to coincide very closely with the red, but it must be remembered

that the origin of coördinates is a long distance below the paper, and the apparent differences are but a very small fraction of the whole. The unit is the tenth meter for wave-lengths.

Having these constants, they may now be substituted in (19), and c determined from the observed values of Verdet's constant. These observed values are: —

Line of spectrum. λ.	Verdet's constant for carbon bisulphide. v.
C 6563	0.0319′
D 5893	0.0415′
E 5270	0.0537′
F 4861	0.0667′
G 4308	0.0920′

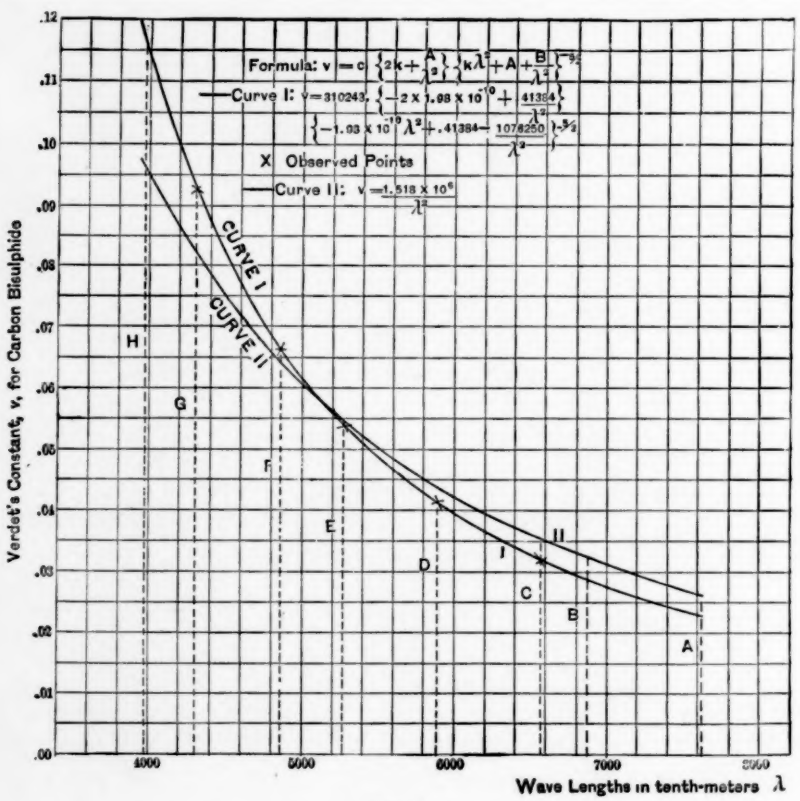

Fig. 5.

The constant thus determined, where a minute is the unit of angle, gives

$$c = 310243. \tag{23}$$

Using these constants for equation (19), we obtain curve I. in Fig. 5. The points marked × are observed values, and the calculated curve practically passes through them all.

The curve II. in this diagram represents the approximate law that Verdet's constant is inversely as the square of the wavelength, and the degree of the approximation may be observed.

MINOR CONTRIBUTIONS.

A NEW FORM OF SPECTROPHOTOMETER.

BY EDWARD L. NICHOLS.

IN 1890 the writer described a form of spectrophotometer, the characteristic feature of which was the use of a horizontal, instead of a vertical, slit.[1] This instrument, which was intended for use upon the photometer bar, I called the horizontal slit photometer. It is shown in Figs. 1 and 2, which are taken from a model constructed since the writing of the paper just cited.

Recently further modifications have been made, and certain desirable features have been introduced. These it is the purpose of this note to describe.

In its original form the horizontal slit photometer is a direct vision instrument, in which the requirements of symmetry are carefully met. The

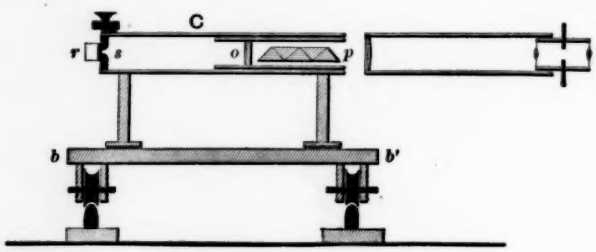

Fig. 1.

light is introduced to the right and left halves of the slit by means of a pair of rectangular prisms for total reflection, one of the acute edges of each of which has been ground away so as to give two surfaces about 2 mm. in width (see Fig. 3). These narrow ground faces are placed together, as shown in the figure, in a plane which bisects the slit transversely; their image forms the boundary between the two spectra in the field of view. Thus in the method of introduction into the collimator tube, the two rays of light are treated in a precisely similar manner. Within the tube, and along the entire path to the eye, likewise, identity of treatment is main-

[1] Trans. of the American Institute of Electrical Engineers, Vol. 7.

tained. This is a point in which many forms of spectrophotometer are at fault.

The prismatic spectrum, although superior to normal spectra as regards brightness, is objectionable for the purposes of spectrophotometry, be-

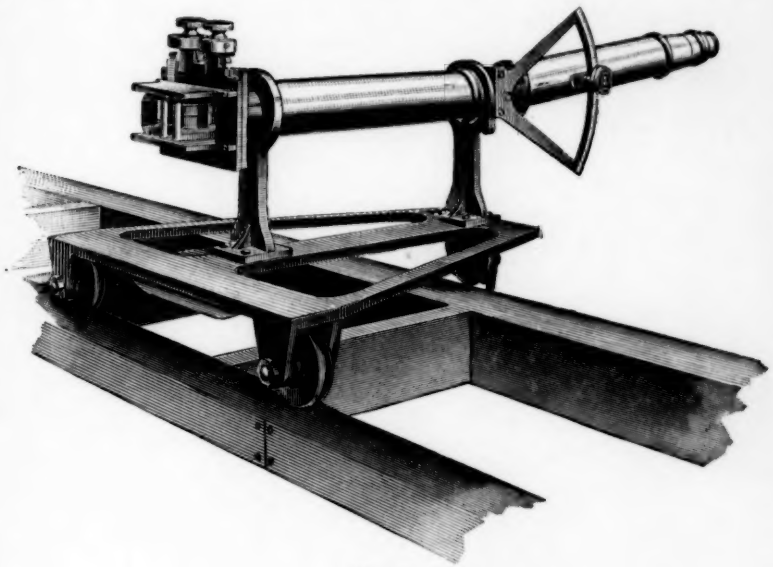

Fig. 2.

cause the less refrangible rays are crowded together. In the orange-yellow and yellow-green particularly, uniformity of tint cannot be secured unless the field of view be made extremely narrow, or high magnifying power be resorted to. In prisms of the densest flint, moreover, the violet is very dim, owing to the strong selective absorption in that region. The diffraction spectrum is free from these peculiarities.

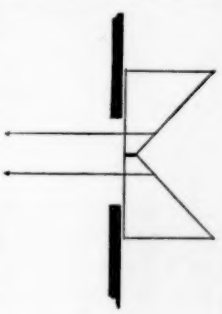

Fig. 3.

The modification of the horizontal slit photometer to be described here contains a grating instead of a prism, and it possesses the further advantage that it brings the slit, with whatever accessory apparatus may be selected for the adjustment of the spectra under comparison to equal brightness, within easy reach of the experimenter. The form of the instrument is shown in Figs. 4 and 5, the latter of which is a simple diagram of the essential parts. The collimator tube is mounted rigidly in a horizontal

position upon a massive base. It carries a Vierordt by means of which the brightness of the two spectra may be adjusted.

Four years of constant use have verified my opinion as to the advantages of the horizontal slit in spectrophotometry: accordingly in the new instrument the slit is placed horizontally and the light is introduced by means of a pair of reflecting prisms such as have been indicated in Fig. 3. The observing telescope swings between a pair of vertical arcs which are rigidly attached to the base of the instrument, the axis of rotation being a hori-

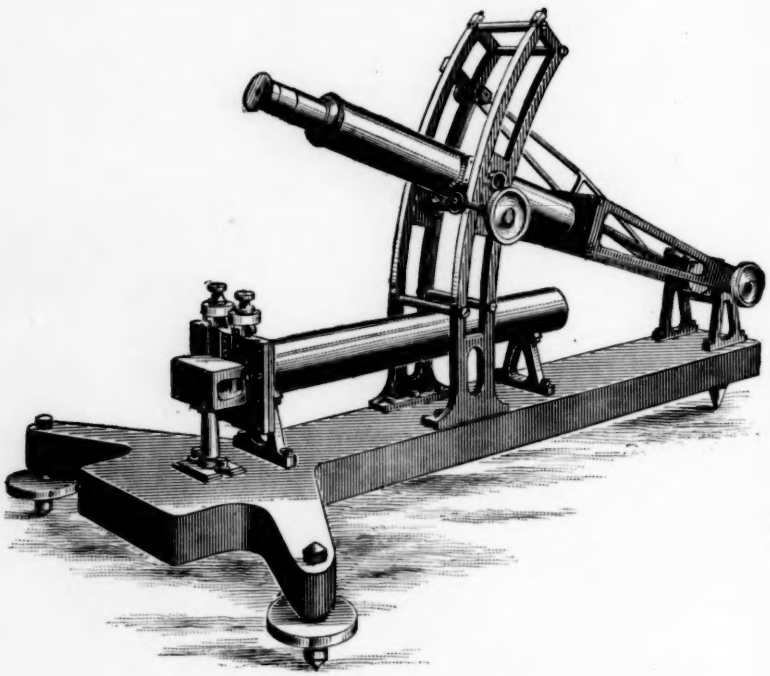

Fig. 4.

zontal line which cuts the optical axis of the collimator perpendicularly at a distance of about twenty centimeters from the objective of the latter. In an adjustable holder swinging in this same horizontal axis is mounted a plane Rowland grating which, receiving parallel rays from the collimator tube, produces two parallel sets of vertical diffraction spectra. The grating and the observing telescope being free to move independently about the same horizontal axis, any portion of any spectrum may be brought into the field of view. After adjustment the grating is held in position by means of a light arm which moves along the face of one of the sectors

already described. The two sectors are provided with scales; the graduation of each is in equal parts.

This form of horizontal slit photometer, which, to distinguish it, may be called the *vertical arc* pattern, may be mounted upon a carriage and used in the photometer room like the earlier form, or two pairs of Nicol's prisms may be mounted to the right and left hand of the slit, by which means the spectra may be brought to equal brightness, or, best of all, where the character of the work will admit of such a method, the brightness of the two sources of light, the spectra of which are to be compared, may be adjusted by means of micrometer slits placed between them and the collimator slit. The use of such slits in combination with the ordinary Vierordt slit, which, taken by itself, can only be used to advantage between very narrow limits, gives a sufficient range, together with great simplicity of action, and the freedom from the errors to which polarizing spectrometers are subject. The method is applicable, however, only in cases in which the sources of light can be made to fill the field of these adjustable slits completely. Where this is possible this method is one of the most satisfactory of those which are available in spectrophotometry.

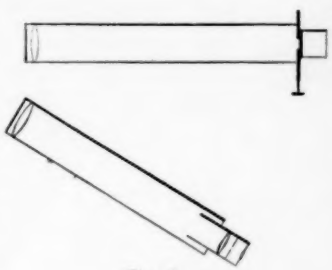

Fig. 5.

A Phonographic Method for Recording the Alternating Current Curve.

By C. J. Rollefson.

In this paper it is intended to give only a short description of a method devised by the writer for the study of alternate current phenomena. After a number of devices had been tried, the use of the phonograph suggested itself. The phonograph records vibrations of sound, which are similar to the vibrations produced by an alternating dynamo, and reproduces the quality of the sound to such a high degree of accuracy that it seems that it might prove an accurate instrument for recording the alternating current curve.

The method, in brief, consists of two operations: first, a record of the curve must be produced on the wax cylinder of the phonograph; second, the indentations produced on the cylinder in the first operation must be magnified by means of a suitable multiplying arrangement.

A section of the arrangement for producing a record of the curve on the wax cylinder is shown in Fig. 1. A small steel magnet, *a*, is fastened to the center of the glass diaphragm, *b*. Around a small bundle of fine iron wires, *c*, are wound a few turns of insulated copper wire, the small electro-magnet thus made being fastened in front of the small steel magnet attached to the diaphragm. Turning the cylinder, *C*, of the phonograph, and passing the alternating current through the coil of the electro-magnet, the small steel magnet, *a*, is alternately attracted and repelled, and the vibrating motion thus produced transmitted to the diaphragm, *b*, and the sapphire-arm, *l*, and thus indentations corresponding to the vibratory motion of the small steel magnet are produced on the wax cylinder.

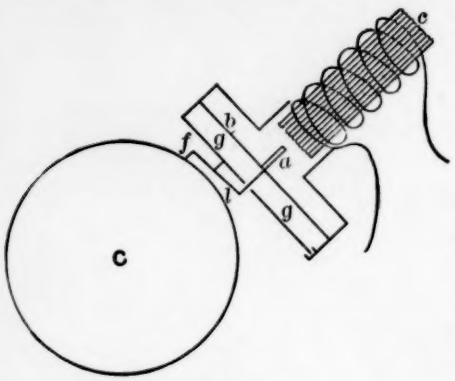

Fig. 1.

The first operation is now complete; it remains to magnify the indentations produced on the wax cylinder. A section of the apparatus which has proven successful for this purpose is shown in Fig. 2. Two small brass bars, *BB'*, are taken, the lower one being fastened rigidly to the diaphragm arm (not shown in the figure), and the upper being free to move. A rubber band, *n*, passing around the two bars holds the upper bar in position. Between the bars are placed two small rollers of very fine iron wire. To the end of one of the rollers is attached a small mirror, *m*, so that

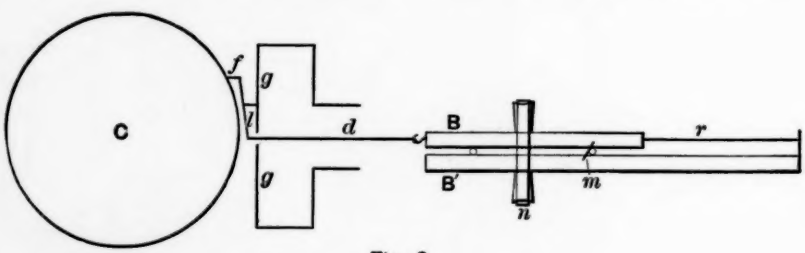

Fig. 2.

readings may be taken with a telescope and scale. The glass diaphragm is removed, and a fine wire, *d*, fastened to the sapphire-arm, *l*, and to one end of the movable bar, *B*. At the other end of the movable bar is fastened a rubber band, or spring, *r*, which serves to keep the reproducing-

stylus, *f*, pressed against the wax cylinder. The reproducer-weight, *g*, to which the sapphire-arm is attached, is now fastened; otherwise this weight would move, and the movable bar remain stationary. The wax cylinder, *C*, is then turned successively through a very small angle by means of an arrangement not shown in the figure, and readings taken.

Only a few preliminary experiments have been performed, but the results obtained seem to be very promising. That the curve should be absolutely perfect is, of course, not to be expected, when inertia must be taken into account. The writer believes, however, that inertia does not play an important part in this method. The movement of the glass diaphragm when the curve is recorded on the wax cylinder may be made so small that the indentations produced on the cylinder are scarcely visible to the naked eye. In the second operation inertia does not enter, as there is no motion of any of the parts when the readings are taken.

This method seems to be especially adapted for the study of harmonics in the alternating current. By the aid of König's apparatus the number of harmonics present may be determined, and, perhaps, their relative intensities. Then, by plotting the curve, their relative positions may be determined. These operations may be repeated several times, and thus each operation serve as a check on the others.

The writer expects to continue investigations along this line the coming year.

ITHACA, N.Y., June 30, 1894.

ON THE RESISTANCE OFFERED BY A MEDIUM CONTAINED BETWEEN SPHERICAL SURFACES AND THEIR MERCATORIAL PROJECTIONS.

BY EDWIN J. HOUSTON AND A. E. KENNELLY.

IT may have escaped notice that a curiously simple relation exists between the resistance of a medium measured between concentric spherical surfaces and the resistance of the same medium between co-axial cylindrical surfaces.

CASE I.

Thus let a and A (Fig. 1) be the radii, in centimeters, of two concentric spherical shells, enclosing between them a homogeneous isotropic medium of resistivity ρ (thermal, electrostatic, electric, magnetic, or radiant);

then if R denotes the resistance of the medium between the spherical surfaces, we know that

$$R = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{A}\right).$$

If, however, Fig. 1 be taken to represent the cross-section of two coaxial cylinders instead of two concentric spheres, then the resistance R', between the cylinders per unit length of both is

$$R' = \frac{2\rho}{4\pi}\left(\log\frac{1}{a} - \log\frac{1}{A}\right);$$

or, as is usually written,

$$= \frac{\rho}{2\pi}\left(\log\frac{A}{a}\right),$$

from which it will be seen that the resistance per unit length of cylinders is obtained by doubling the "spherical resistance," after the terms within the brackets have been replaced by their respective natural logarithms.

CASE II.

Let a and A (Fig. 2) be the sections of two spheres of radii a and A cm., situated at relatively large inter-central distances d cms. Then the resistance between them will be

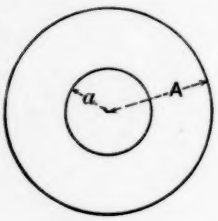

Fig. 1.

$$R = \frac{\rho}{4\pi}\left(\frac{1}{a} + \frac{1}{A} - 2\frac{1}{d}\right).$$

If, however, a and A be regarded as the cross-sections of two indefinitely long parallel cylinders of radius a and A, separated by an interaxial distance d, the corresponding resistance between them per centimeter length of both is

$$R' = \frac{2\rho}{4\pi}\left(\log\frac{1}{a} + \log\frac{1}{A} - 2\log\frac{1}{d}\right);$$

or, as usually written,

$$R' = \frac{\rho}{2\pi}\log\left(\frac{d^2}{Aa}\right),$$

from which it will be seen that R' is again derived from R by doubling the value of R after the quantities in the brackets have been replaced by their respective logarithms.

When, in Case I., the spheres are not concentric, or when, in Case II.,

they are not small relatively to their intervening distance, the resistance between spherical electrodes becomes more difficult to compute, and the above simple relationship with the resistance between cylindrical electrodes is no longer retained. But the resistance between cylindrical electrodes can always be computed by simple formulæ for any eccentricity or intervening distance, and if any comparatively simple relation could be

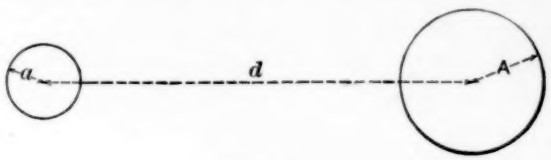

Fig. 2.

established between them and the corresponding spherical conditions by generalized formulæ, the application would be useful in practice. We have not succeeded in discovering this relationship for the general case, but there probably is, nevertheless, a comparatively simple one, since some of the numerical values calculated by Kelvin for resistance between spherical electrodes in close proximity, and published more than twenty years ago in the *Philosophical Magazine*, display suggestively simple relations at the critical values of the surd quantities in the corresponding cylindrical formulæ.

NEW BOOKS.

A History of Mathematics. By FLORIAN CAJORI, Professor of Physics in Colorado College. New York, Macmillan, 1894.

If history can be trusted, Alexander the Great was told by his tutor, Menæchmus, that there was no royal road to geometry (mathematics). But Menæchmus lived two thousand years ago.

The modern student who is acquainted with the ponderous syllogisms and awful notation or lack of notation of the ancients, is quite willing to admit that the modern road is royal and more than royal, and probably Menæchmus would agree with him.

Only one acquainted with the horrors of the "middle passage" from the ancient mathematical servitude, can be really appreciative of the modern release from the shackles of ancient conservatism and neophobia, and of the intellectual greatness of the men who could work with such tools as did our predecessors.

It is only when the student runs across some such expression as

$$x^{\bar{\varepsilon}}\,\bar{a}\,\bar{\varsigma}\,\bar{\varsigma}\,\bar{\eta}\,\pitchfork^{\iota}\,\delta^{\circ}\,\bar{\epsilon}\,\mu^{\delta}\,\bar{a}\,\iota\,\varsigma\,\bar{a},$$

which Diophantus used in place of our $(x^3+8x)-(5x^2+1)=x$, or

$$\text{Ꝝ v. cu. Ꝝ } 108\ \bar{\text{p}}.\ 10\ |\ \bar{\text{m}}\ \text{Ꝝ v. cu. Ꝝ } 108\ \bar{\text{m}}\ 10,$$

which Cardan, as late as 1545, used for $\sqrt[3]{\sqrt{108}+10}-\sqrt[3]{\sqrt{108}-10}$, or

$$^{25}\,⓪\,3\,①\,7\,②\,9\,③,$$

used by Stevinus, as late as 1585, for 25.379, that he begins to understand the awful slough through which his predecessors have floundered and to appreciate "others have labored, and ye are entered into their labor."

Nesselmann recognized three stages in this floundering from ancient to modern nomenclature which he designates as the *Rhetorical*, the *Syncopated*, and the *Symbolical*. The *Rhetorical* method determined the unknown quantity by a long course of logical reasoning expressed in words without the use of any symbols whatever. The abbreviation of words of frequent occurrence gave rise to the *Syncopated*, which was gradually superseded by the modern *Symbolical* method.

Curiosity prompts the question, when and where did our present mathematical tools take shape; and gratitude, by whom were they invented. It

is interesting, in fact necessary, for the mathematician to know the shoals and quicksands upon which his predecessors have foundered. Such knowledge would have saved the labor wasted by "the great army of circle squarers" who have "for two thousand years been assaulting a fortification which is as indestructible as the firmament of heaven."

It is a revelation to the reader to learn how

Equality was expressed by ι, Diophantus (ob. 330); by =, Ricorde (1540); by ||, Xylander (1575), though generally written at length till the 1600's; by ∝, ∞, a contraction of *æqualis* (1600–1680); by ::, Oughtred (1631) and Wallis (1686).

Inequality by >, <, Harriott (1631); by ⌐¬, ⌐_¬, Oughtred (till ± 1700).

Addition by juxtaposition, Greeks and Hindoos; by *P*, *p*, Italians; by *e*, Pacioli (1494); by ϕ, Tartaglia (ob. 1557); by +, for excess, in 1489, and again in 1544, and generally in 1660.

Subtraction by ℽ, Diophantus (ob. 330); by ·, Hindoos; by *M*, *m*, Italians; by *de*, Pacioli (1494); by −, (± 1630).

Multiplication by ×, Oughtred (1631), Harriott (1631); by ·, Descartes (ob. 1650); by ⌢, Leibnitz (1686).

Division by $a-b$, a/b, $\frac{a}{b}$, Arabs; by ·, Oughtred (1631); by ÷, Pell (1630), previous to which it had been $\div\!\!\cdot$.

Powers by ⌣, $\overset{1}{\smile}$, $\overset{2}{\smile}$, etc., Bombelli (1579); by ①, ②, ③, etc., Stevinus (1586); *Aq*, *Ac* for A^2, A^3, etc., Vieta (1591); *aa* for a^2, *aaa* for a^3, Harriott (1631); *a* 2, *a* 3 for a^2, a^3, Herigone (1634); a^2, a^3, etc., Descartes (1637); x^{-1}, $x^{2/3}$, etc., Wallis (1659); a^n, Newton (ob. 1727).

Roots by √, Rudolff (1525); ww for 4th root, Rudolff and Schurbel (1551); ww for $\sqrt[3]{}$ by the same; $\sqrt{3}$, $\sqrt{\sqrt{33}}$ for $\sqrt{}$, $\sqrt[4]{}$, by Stifel (ob. 1567); $\sqrt{2}$, $\sqrt{3}$ for $\sqrt{}$, $\sqrt[3]{}$ by other writers; ⌊10, ⌊_10, ⌋⌊ 10 for $\sqrt{10}$, $\sqrt[3]{10}$, $\sqrt[20]{10}$ by Napier.

He gets a new idea of the progress and mutations of mathematics as he views the transitions from

1 Zp · 5 Rm 4
1 Q + 5 N − 4, Xylander (1575).
$1^{\overset{2}{\smile}}p\ 5^{\overset{1}{\smile}}m\ 4$, Bombelli (1572).
1 ② + 5 ① − 4 ⓪, Stevinus (1586).

to $x^2 + 5x - 4$;

and from 25 ⓪ 3 ① 7 ② 9 ② Stevinus (1585),
25, 3′ 7″ 9‴ Stevinus (1585),
25 $\underline{379}$ Briggs (ob. 1631),
25 | $\underline{379}$ Later writers,

to 25·379 Napier (ob. 1617).

His respect for the inventors and early users of calculus will not be lessened by comparing the

$$\overline{\text{omn. } \overline{\text{omn. } l}\,\frac{l}{a}} = \frac{\overline{\text{omn. } l^2}}{2a}$$

of Cavalieri with

$$\int\int l\,\frac{l}{a} = \frac{\int l^2}{2a}.$$

All this and more will be found delightfully set forth in Cajori's *History of Mathematics*. In many points the book cannot be praised too highly, especially the portion which treats of the progress of mathematics in recent times.

Taken as a whole, we may say that it is the best text-book on this subject which has yet appeared; however, one cannot but regret that the author did not treat the subject on somewhat different lines, making it more truly a history of mathematics rather than a history of mathematicians. More diagrams illustrative of the problems referred to in the text would have increased the value of the book to students. The majority of mathematical readers would be more interested to know that the first curve rectified was a cubic parabola in 1660, the second a cycloid in 1673, that the polar triangle was introduced in the early 1600's, as also the word *focus*, than to know primarily by whom.

The author's explanation of the derivation of the word *algebra*, from *aldshebr walmukabala* (restoration and reduction, transposition and combination), is clear and interesting. The derivation of *sine*, from the Latin *sinus*, a translation of the Arabic word *dschaib*, which was mistaken for its homonym *dschaiba*, would have been improved if *jiva* the Sanscrit, original of *dschaiba*, had been explained to be a *bowstring*.

The description of the invention of logarithms would be clearer to students (for whom the book is intended) if attention had been called to the principle upon which Napier based his reasoning; viz.: In a geometrical progression, the terms of which are *numbered consecutively* (any term of the progression being found by multiplying together two previous terms and dividing by the first term of the series), *if* we add the *numbers* corresponding to the multiplier terms, and subtract that corresponding to the divisor term, the result will be the number corresponding to the quotient term.

Napier saw that if he could establish two corresponding series, the process of multiplication could be replaced by that of addition, especially if the first terms were respectively 1 and 0, as in the series

1	2	4	8	16	32
0	1	2	3	4	5,

in which case the operation of division and its correlate, subtraction, would be dispensed with.

To construct the two series, Napier conceived two points to move from the initial position with the same initial velocity, one at a constant velocity, the other at a velocity proportional at any instant to the space yet to be passed over.

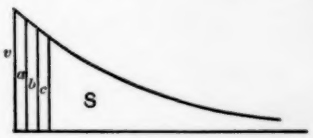

Fig. 1.

The space described by one would increase in arithmetical progression, the other decrease in geometrical progression.

In Fig. 1, let the ordinates represent the velocities, v the initial velocity, and a, b, c ··· the increments of space at successive instants of time, and S the whole space to be described from the initial point.

From the method of generation

$$\frac{a}{b}=\frac{S}{S-a}, \text{ or } \frac{S}{a}=\frac{S-a}{b},$$

which by division becomes

$$\frac{S}{S-a}=\frac{S-a}{S-a-b}.$$

Similarly,

$$\frac{S-a}{S-a-b}=\frac{S-a-b}{S-a-b-c},$$

and so on. That is, the increments are proportional to the remaining space. Q. E. D.

Napier assumed the space S as 10^7, or what is the same thing, since the ordinates are proportional to the space s, $v=10^7$, and proceeded to calculate the successive ordinates (geometricals) and the corresponding arithmeticals.

The ordinates might as well have been increasing, and the initial one equal to unity, in which case we should have as the equation of the curve $y=e^x$, the curve of the natural system of logarithms.

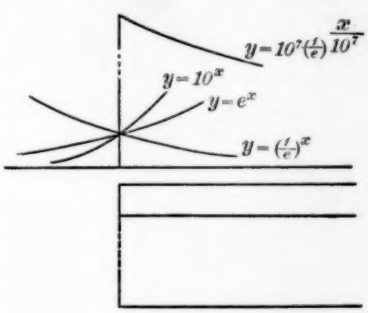

Fig. 2.

The different systems are shown in the second diagram, where $y=e^x$, $y=10^x$, $y=\left(\frac{1}{e}\right)^x$, $y=10^7\left(\frac{1}{e}\right)^{\frac{x}{10^7}}$ represent respectively the natural, common, inverse, and Napierian systems.

The lower band represents the equicrescent areas corresponding to the

areas in geometrical progression in the upper part of the diagram. This band is double, one for areas generated with unit initial velocity, and the other for areas generated with 10^7 initial velocity. It is evident from the figure that any multiple of a system of logarithms is a new system of logarithms.

To get the Napierian curve, consider that the areas of the Napierian curve are 10^7 times the corresponding areas of the inverse curve $y = \left(\frac{1}{e}\right)^x$. Then

$$Y = \text{an area of the Napierian curve}$$
$$= 10^7\left(\frac{1}{e}\right)^x = 10^7\left(\frac{1}{e}\right)^{\frac{N.L.}{10^7}},$$

the arithmetical area x being divided by 10^7, since the Napierian arithmeticals are 10^7 as large as the arithmeticals of the inverse system.

The relation between the different systems is shown by

$$y = e^{\lambda} = \left(\frac{1}{e}\right)^{\lambda} = 10^{l} = 10^7\left(\frac{1}{e}\right)^{\frac{N.L.}{10^7}},$$

where λ = natural logarithm of y, l = common logarithm of y, and $N.L.$ = Napierian logarithm of y.

From these equations we easily get

$$\lambda = \text{nat. log } y = \text{nat. log } 10^7 - \frac{N.L.}{10^7},$$

whence

$$N.L. = \text{Nap. log } y = 10^7(\text{nat. log } 10^7 - \text{nat. log } y)$$
$$= 10^7 \text{ nat. log } \frac{10^7}{y}.$$

In explanation of the word *logarithm:* In a series of magnitudes and their logarithms, the logarithm corresponding to any magnitude indicates the number of geometrical ratios necessary to produce the magnitude from unity; *e.g.:*—

1	2	4	8	16	32	. . . .	magnitudes,
0	1	2	3	4	5	. . . .	logarithms,
	$\frac{1}{2}=\frac{2}{4}=\frac{4}{8}=\frac{8}{16}=\frac{16}{32}$					. . . .	ratios,
0	1	2	3	4	5	. . . .	number of the ratio;

or

1	10	100	1000	10000	. . . . magnitudes,
0	1	2	3	4	. . . . logarithms,
	$\frac{1}{10}=\frac{10}{10^2}=\frac{10^2}{10^3}=\frac{10^3}{10^4}$				. . . . ratio,
0	1	2	3	4	. . . . number of the ratio.

If we take 10^5 ratios between 1 and 10, then $\log 10 = 10^5$, and if we should form the series of ratios, we should find that 2 would be the consequent of the 30,103d ratio, and 3 of the 47,712th ratio, etc. That is, 30,103 would be the *number of the ratio* (λογος αριθμος, logarithm) corresponding to 2, and 47,712 that corresponding to 4, etc.

If we had taken 1 as the logarithm (λογος αριθμος) of 10, the logarithms of 2 and 3 and of all the other magnitudes would have been correspondingly smaller; viz. 0.30103, 0.47712, etc., as in the common system.

The author adheres to the term *centrifugal force*, which should be discarded or else accompanied by an explanatory plea of "confession and avoidance." He likewise uses *center of gravity* where he means *center of inertia*, regardless of the fact that comparatively few bodies have a center of gravity.

Frequent references to publications are given where the reader can find the material to satisfy the curiosity aroused by the text. The citations in the chapters on modern mathematics are particularly valuable, though not as numerous as they might have been.

If I may venture a friendly criticism, the author frequently fails to keep his subject down to the level of his readers.

For example, his reference to *Coss* and *Cossists* would be immediately intellegible had he added parenthetically that *Cosa* was the Italian for *thing*, the algebraic designation for the unknown quantity at that time.

His references to the combinatorial school, quadratic reciprocity, quadratic residues, etc., would have been more readable had there been a short explanation of their meaning accompanying them. A few additional lines would accomplish this. A certain knowledge of terms in current use, such as determinant, potential, Abelian function, Hessians, Pfaffians, etc., must be presupposed; but historical terms, if we may so designate them, should be explained in a history of mathematics, if anywhere.

This is the nearest approach to a text-book on this subject for class-room use that we have, and so far as it goes is excellent. But a rearrangement on the lines indicated would, I think, make it much more available. The history of mathematics has been too much neglected in our schools and colleges, and it is to be hoped that the appearance of this book may result in the introduction into the curriculum of this much neglected and sorely needed branch of mathematics. As the author says on his title page, "no subject loses more than mathematics by an attempt to dissociate it from its history."

The present work should be read by every student who loves high thinking and desires to know something of that science which makes the greatest demands on the intellectual forces of its votaries, and which has taxed to the utmost the mightiest minds that the world has produced.

Some will be scandalized to learn that Euclid's famous 12th axiom may perhaps be no axiom, and that possibly space itself may be finite; that the sum of the angles of a triangle may not be a straight angle, after all.

Many, too, will learn for the first time that mathematics is not a dead science, completed and mummified by Euclid and Newton, but that under the creative hands of Gauss, Riemann, Lobatchewsky, Grassmann, and of Sylvester, Cayley, Steiner, Klein, and a host of others, it has a vitality which promises as much for the future as has been done in the past.

It will be found to be a most readable and enjoyable book by every one who is interested in the intellectual progress of the race, and particularly by those who desire to know something of what is going on at the present day in the world of rigid scientific thought.

Like every new book it has some errors and misprints, which will doubtless be duly corrected.

ARTHUR LATHAM BAKER.

UNIVERSITY OF ROCHESTER,
April, 1894.

Electromagnetic Theory. Vol. I. By OLIVER HEAVISIDE, F.R.S. pp. xxi + 466. New York, Van Nostrand Company.

This work is a reprint of a series of papers which originally appeared in the London *Electrician* during the years 1891–93. It does not postulate so extensive a knowledge of mathematics as do the author's *Electrical Papers*, but it addresses all earnest students who have been trained to think with some degree of exactness. All such will find in it a valuable storehouse of ideas. The book is divided into four chapters, of which the first is introductory; the second presents an outline of the electromagnetic connections; the third expounds the elements of vectorial algebra and analysis; while the fourth gives the theory of plane electromagnetic waves.

In the introductory chapter some current delusions on the subject of mathematical investigation are exposed; for instance, that mathematics is a waste of time, or that it is a mere machine which can grind out only what is put in. The latter view has been advocated by pretentious philosophers, such as Mill and Huxley; in this they merely demonstrate that they have failed to understand the nature of exact analysis, the finest part of human knowledge. Mr. Heaviside takes the view that mathematics is reasoning about quantities. He says: "Even if qualities are in question, it is their quantities that are subjected to the mathematics. If there be something which cannot be reduced to a quantity, or, more generally, to a definite function, no matter how complex and involved, of any number of other quantities which can be measured (either actually or in imagination),

then that something cannot be accurately reasoned about, because it is in part unknown." This view will appear too narrow to the pure analyst, probably even to the mathematical physicist, for has he not to consider identity and relation as well as equality?

The outline of the fundamentals of electromagnetic theory given in the second chapter is characterized by the use of rational units, vector analysis, and the duplex form whereby the electric and magnetic sides of electromagnetism are symmetrically exhibited and connected. "The forces and fluxes are the objects of immediate attention, instead of the potential functions, which are such powerful aids in obscuring and complicating the subject, and hiding from view useful, and sometimes important, relations."

In the electromagnetic system of units the unit magnetic pole is defined by means of the equation $f = mm'/r^2$. According to Mr. Heaviside f is not equal to the above function, but becomes equal when the function is divided by $4\pi c$, where c denotes the permittivity of the dielectric. If c be taken as 1, the equation becomes $f = mm'/4\pi r^2$. He shows that if the unit of quantity be defined by this last equation, then the electric and magnetic equations become not mere formulæ, but rational truths with transparent meaning. In this connection I may mention that, when writing *Physical Arithmetic*, I found that almost all the physical coefficients could be expressed in plain English, with the help of *per* and *by* and an occasional bracket, but that I encountered a difficulty in thus rationalizing the law of the inverse square. My solution (p. 298) was that the strength of a source is properly expressed by so many units of current per unit of solid angle, the unit of solid angle being defined after the analogy of the radian. This agrees precisely with our author's view. The existing system of units involves the assumption that the proper unit of solid angle is the ratio of the whole spherical surface to the square of the radius; that is, the steregon. But as the radian, not the whole circumference or the quadrant divided by the radius, is the proper unit of plane angle, so the steradian — that is, unit of spherical surface per square of the radius — is the correct unit of solid angle. The word *steradian* was introduced by Dr. Halsted thirteen years ago in his *Metrical Geometry*. There can be no question about the superiority of the rational units, but it does not follow that the legalized units ought to be changed immediately; for may not further light bring to view further desirable changes?

The third chapter expounds the elements of vector analysis. The exposition, while clear, and suitable for the class of readers addressed, contains principles which appear of doubtful validity to those who have made a special study of the matter. For instance, the scalar product of two vectors is not distinguished by any prefix, though the vector product is, apparently on the same principle that the eldest daughter is sufficiently

distinguished by Miss, provided all the younger sisters have their Christian names appended. But the scalar product and the vector product are only partial products; the simpler name belongs properly to the complete product, which is their sum. Here we have an indication of one of the principal defects of the analysis; it is fragmentary, not a method *totus teres atque rotundus*. Again, the author founds his addition of vectors on the composition of successive translations. This plan, though usually adopted by writers on vector analysis, is inadequate, for it does not apply *simpliciter* to such vectors as forces applied at different points. Rotations are compounded in that manner, provided they are infinitesimal; to compound finite rotations requires ideas more complex than the simple vector.

The fourth chapter, devoted to the theory of plane electromagnetic waves, is mainly descriptive, and may be read with profit by many who are unable to follow the mathematical investigation from which the results are derived. The theory is applied to straight wires, and an account is given of the effects of self-induction and leakage, and the application shown to the problem of long-distance telephony.

ALEXANDER MACFARLANE.

Theory of Heat. By THOMAS PRESTON. 8vo, xvi + 719 pp. London, Macmillan, 1894.

Professor Thomas Preston's *Theory of Light* appeared in 1890, and has proved itself a serviceable book. It is now followed by his *Theory of Heat*. Such titles are significant of the stage upon which the more advanced work in physics has entered. They mark the attempt to weld together the results of modern research and those which have long stood as classical; to present a connected digest of established fact, and of theoretical doctrine which is on the safe side of the frontier of speculation. The public demand which these books imply, is one sign of a lively interest in the completer study of physics.

Some books of this type presuppose readers who are already familiar with elementary phases of the subject; a well-known instance is Lord Rayleigh's *Theory of Sound*. But the volume now before us concedes 100 pages of "Preliminary Sketch" to the needs of beginners, and demands only maturity of mind, with some knowledge of calculus, of those who are to follow its arguments.

The plan may have practical grounds, such as a desire to make the book available for a larger circle of readers, but it lies open to several objections, which are only confirmed by the attempt to execute it in the present instance.

It would be easy to point out a wavering conception of the intelligence

of the reader, as shown by the selection of preparatory ideas, and to give examples of repetition, or separation of related matters, or "preliminary considerations" which are not followed up; but these would be minor criticisms; the cardinal defect is that the argument of the book is broken up, without any guarantee that the parted links form a complete chain. If the reader be really beginning the subject, he is either confused by unwarranted assumptions as to his fundamental ideas, or, more dangerous still, led to overlook the blank spaces in his own knowledge.

The truth is, that with the current opportunities for laboratory instruction, nobody should make the first approach to physics by reading merely; while if work has been done in a laboratory, many a precious page in a book like this, now devoted to A B C explanation, might be reserved to fill more serious gaps.

The work must, therefore, be judged from the standpoint of a student with some preparation. Such an one will find a part of the material offered already familiar, it may be, but he will recognize a freshness of treatment in it; and he will find a stimulus and an aid for the prosecution of his study in the references to literature which are liberally given throughout. There are, perhaps, a thousand of these in all; they include journals in English, French, and German, and more recent dates are *not* conspicuous by their absence. The German workers are least fairly represented; references to papers in that language form about one-sixth only of the total number, while the division is nearly equal between English and French. No one can expect such references to be uniform in value or novelty; but it is surprising that the author finds it advisable to refer us (p. 8) to Tait's *Sketch of Thermodynamics* for the nature of the candid scientific mind.

Although our author recognizes the necessity of selection in a case like this, and has, for reasons which he states, omitted several topics, his volume has, nevertheless, swelled to over 700 octavo pages. It therefore supports the protest of its preface against "pocket editions of the sciences." Not counting the preliminary sketch already spoken of, the book is equally divided between a group of five shorter chapters on Thermometry, Dilatation, Calorimetry, Radiation, and Conduction, and two longer chapters on Change of State and Thermodynamics.

The emphasis of the book is thus indicated as falling upon the energy relations of heat, and the series formed by the three states of matter. And this is quite natural in the days when molecular physics, upon a dynamical basis, seems to be the goal before us.

In all seven chapters the summaries of experimental work are such a prominent feature that this impresses one at the first glance. Historical order is preserved; a useful sifting and analysis of results is given in many

important cases; and the process of scientific development through elimination of error is shown by the discussion of examples.

Notable instances of such work well done are: Chapter V., Section VI., "On the Continuity of State," where the discussion is made to include important later work; and the same chapter, Section VII., "On Equations relating to the Fluid State," in which the present condition of knowledge is worked out from the crude beginning made by Boyle. Minor examples are: "The Caloric Theory and its Overthrow"; "The Development of the Air Thermometer"; "The Radiometer."

The prevailing spirit of fairness is maintained when J. R. Mayer is set in his proper place, and when the attempt is made to purge Carnot of heterodoxy on the Caloric question. But is it intended to stigmatize Balfour Stewart as a "popular expounder" of science (p. 282)? And Maxwell's sorting-demon was hardly a mere "ingenious, but illusory, violator" (of the second law of Thermodynamics): this conception surely contributed to clearness of thought (p. 620).

Professor Preston is rightly a stickler for logical thinking. He is early at work upon the conception of temperature (p. 12), and his attempts to introduce logical order into our notions about thermometers are found throughout the book. But while he starts the fox, he can scarcely be said to run it down. Few will follow him in believing that "what we directly *observe* is temperature" (the author's italics). Do we not, in most cases, observe constancy, or change, of volume in the thermometric substance? Or, in some others, what proves to be a result of rate of transfer of energy?

In fact, it does not seem as if logic, and definition, and the more rigorous dynamical ideas of the subject were the strong points of the book. We are asked to believe (p. 58) that "the property that really is conserved throughout chemical processes" is weight (and not mass). That is, the weight at Panama of some mixed iron filings and flowers of sulphur equals the weight at San Francisco of the compound formed by uniting chemically the constituents of this mixture. Of course Professor Preston does not think this; but our ordinary notions need correction, he says; and this results from the way in which he would correct them. — A definition of molecule: "The smallest part of the body which can be separated from it and still possess all the characteristics which distinguish the substance" (p. 61). Fancy separating one molecule from a body and testing it!

The phrase "vibration of molecules" is used throughout the book. Careful search has failed to show, however, whether this is to be understood as meaning vibration of the molecule as a whole, or of the atoms which constitute the molecule.

Unless we know this, it is difficult to judge quite fairly the view of conduction presented (p. 506). But one is inclined to characterize it as

misleadingly simplified. The tuning-fork analogy will help in some cases; but when we meet it here, the temptation is strong to quote the author's own words about glacier-motion: "To attempt to explain all the phenomena by attributing them to any single action is certainly not reasonable" (p. 283).

As regards the style, the author has constrained an evident tendency to exuberance which shows in the Introduction, Chapter I.; and has given us in the main sober English, as good as we usually find in scientific books. Occasionally, though, the pen slips. In the following examples the English almost obscures the sense. "The value of any hypothesis depends upon its convenience in systematizing observed facts, and to the extent to which it embraces all known phenomena must its utility be estimated" (p. 61). "We know that energy in immense quantities is perpetually passing through space with the velocity of light in the form of radiant heat and light, and that it exists also in the so-called potential state. The former exists in the ether, and probably also the latter" (p. 82).

We may say then that the book seems likely to be of service, like the *Theory of Light*, but more on account of the experimental results that it gives compendiously and accessibly, than because it proceeds with system and connection to build a structure of theory.

If it were rewritten so as to assume the proper basis of dynamical and experimental ideas, it would gain in compactness, and be more likely to secure recognition for permanent value.

FREDERICK SLATE.

UNIVERSITY OF CALIFORNIA,
April 17, 1894.

A Treatise on the Theory of Functions. By JAMES HARKNESS, M.A., Associate Professor of Mathematics in Bryn Mawr College, Pa., late Scholar of Trinity College, Cambridge, and FRANK MORLEY, M.A., Professor of Mathematics in Haverford College, Pa., late Scholar of King's College, Cambridge. 8vo. pp. ix+507. New York, Macmillan & Co., 1893.

The appearance of this treatise almost simultaneously with the larger work of Forsythe marks an era in the history of mathematical publications in England and the United States.

The earlier chapters are designedly elementary, and in their treatment of the subject leave little to be desired. The investigation of the real functions of a real variable in Chapter II. is helpful in paving the way for the complex variable, which occupies so large a share of the authors' attention. In Chapter III. the discussion of the infinitive series is particu-

larly happy. The subject is considered at some length, and, for example in the case of the circle of convergence, is well stated.

The chapter on Abelian Integrals materially increases the value of the book. While, with regard to Chapter VIII., which treats of Double Theta-Functions, one can scarcely do better than to quote the words of the authors themselves contained in the preface: "Chapter VIII. is devoted to a somewhat condensed treatment of double theta-functions, which goes farther than is necessary for our immediate purpose, for the reason that the subject is not very accessible in the English language."

In the treatise, as a whole, the authors have erred on the side of conciseness. Theorems are stated and demonstrated, but the method of proof is so brief that unless one is quite in touch with the subject he will spend much valuable time in supplying the intermediate steps which are involved. Of course this very conciseness would enhance the value of the work to the reader who merely wished to verify a result. But the majority of us remember a conclusion better than the exact method by which it was attained, and it is frequently convenient to find the outline of a proof well filled in.

The arrangement of the type might be improved if the statement of the theorems differed from the demonstrations in appearance. The eye would assist the mind in detecting what was to be proved. In nearly every case the diagrams are exceedingly good, and their helpfulness leads one to wish that they were even more frequent.

The method of giving "References" at the close of each chapter to standard works on the subject which has just been considered deserves special commendation. The book is sure to prove a valuable addition to the library of any physicist.

L. M. PEIRCE.

The Incandescent Lamp and its Manufacture. By S. T. RAM.

Among the most important factors in the success of an incandescent electric light plant is the incandescent lamp; it is also the one of which many electric light manufacturers have the least knowledge. Possibly this state of affairs is largely because of the mystery with which lamp manufacturers have surrounded the factories and the methods of manufacture, but is also due to the fact that the incandescent lamp is not a piece of machinery having wearing parts to be lubricated, adjusted, and repaired, and the accurate operation of which can be readily understood.

Incandescent lamps have, even for those of the same make, complex relations between initial efficiency, efficiency during life, either total, or whatever may be considered as commercial, life, cost of lamps, cost of

power, etc. The consideration of the advisable efficiency of lamps to use in any given case must either be a matter of judgment or must be based on data obtainable only from experiments of a lengthy and costly nature; such data have been obtained, but are not generally known, and are not in the working library of many electrical engineers.

Mr. Ram's work deals largely with the manufacture of the lamps. The methods described give the principles of operation and the result desired, but the apparatus is not generally such as will appeal to an American manufacturer, because it is not to any great degree automatic. It is extremely difficult to manufacture incandescent lamps by means of automatic machinery. Much, however, has been done in this respect, and very little is described by Mr. Ram.

In the direction of information on the economic use of the lamp, Mr. Ram gives some valuable data, but makes no reference to many important articles on this subject published in America by such investigators as Howell, Nicols, Merritt, Thomas, and others. To mention as "the most recently published tests" those of Professor Ayrton, 1892, is apparently an evidence of a lack of knowledge of what is being done in this country.

Mr. Ram states that all lamps should be operated at a standard efficiency, and the C. P. at such efficiency be marked. If such a course were followed, a factory would produce much unsalable stock, as the commercial conditions are that orders are placed for a certain C. P. and voltage, and must be filled, as nearly as may be, as ordered.

The users of a lamp can find much in Mr. Ram's book which will be of great value. He will appreciate the folly of specifying lamps of a uniform cold resistance, as has been done by alleged experts, and also the importance of maintaining P. D. at the lamps.

Any information which will assist to give a truer knowledge of a factor which amounts to 10 to 20 per cent of the total operating expenses of an electric light station must prove of value, and Mr. Ram has prepared a book from which such information is obtainable, and users as well as manufacturers can read and study it to great advantage.

E. P. ROBERTS.

Electrical Engineering Formulæ. By W. GEIPEL and M. HAMILTON KILGOUR. pp. xx+796. London, The Electrician Publishing Company, 1894.

This pocket book marks by its bulk the increase in the mass of material which the modern engineer is compelled to have at his fingers' ends. The volume has the size of page most convenient for a pocket manual, but it

approaches the limit, if, indeed, it has not passed it, at which smallness of page ceases to be a matter of economy.

Geipel and Kilgour's manual is much more than a collection of engineering formulæ. It is, indeed, in small compass a brief compendium of physical units, of data concerning the properties of materials, of mathematics, including trigonometry, hyperbolic functions, the calculus and differential equations, of hydraulics, of heat and its applications to boilers and the making of steam, of heat engines, and of applied electricity.

That portion of the book which deals with electrical engineering itself begins only upon the 543d page, and occupies scarcely one-third of the volume, so that the saying, attributed to Kelvin, which has done so much to bring into contempt in the minds of a certain class of electrical engineers the very subject with which primarily they have to do, — viz. that electrical engineering is nine-tenths mechanical engineering, — although not strictly verified in the distribution of materials in this work, finds some justification therein. The selection of material, aside from the introduction of so many data which lie within the province of the steam engineer, rather than of the electrician, appears upon the whole to be a judicious one. The tabulated matter is well arranged and accurate.

A glance at the tables indicates that progress is being made among electrical engineers in Great Britain towards the adoption and use of the metric system. It is encouraging to find in a pocket book designed for engineers, temperatures stated in degrees Centigrade as well as in the scale of Fahrenheit, and to find the centimeter taking its place alongside the inch. The necessity of duplicating data in order that those who have not yet accustomed themselves to the C. G. S. system may be able to use this manual, is one which we hope will disappear before many editions have been sent out. Another sign of the times is the disappearance altogether of the department of sub-marine cable testing. It is not many years since this was the chief subject with which British electrical engineers had to deal. In the present volume the cable is treated purely as a device for the transmission of energy in electric lighting and power circuits. Cables for signaling, either by telegraphy or by means of the telephone, are not given a place. The work is one which will be found useful in the hands of practical men on both sides of the Atlantic.

E. L. NICHOLS.